Optimal Design of Research Contests

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Procurement of an innovation often requires substantial effort by potential suppliers. Motivating effort may be difficult if the level of effort and quality of the resulting innovation are unverifiable, if innovators cannot benefit directly by marketing their innovations, and if the buyer cannot extract up-front payments from suppliers. We study the use of contests to procure an innovation in such an environment. An auction in which two suppliers are invited to innovate and then bid their prizes is optimal in a large class of contests. If contestants are asymmetric, it is optimal to handicap the most efficient one. (*JEL* C70, D44, D89, L12, O32)

In 1714, the British Parliament offered a prize of £20,000 for a method of determining longitude at sea to within one-half of a degree. The offer was made following a series of maritime disasters, which included the loss of four warships and nearly 2,000 lives in a single incident. It was already known how to determine latitude at sea, but a simple method for determining longitude remained elusive. A clockmaker named John Harrison solved the problem by developing an accurate portable timepiece.[1] Harrison received an initial payment of £10,000 from the Longitude Board, which oversaw the contest. He received £8,750 more, following an Act of Parliament, 11 years after the first successful trial. The delay resulted in part from skepticism as to whether the successful trial was reproducible.[2] Nonetheless, this contest led to the development of a new generation of portable timepieces (see Dava Sobel, 1995).

Over the years, contests have played a major role in the procurement of many other innovations. In 1829, the Liverpool and Manchester Railway offered a prize of £500 for the best design for an engine to provide passenger service between the two cities (see Curtis Taylor, 1995; Richard Fullerton and R. Preston McAfee, 1999). The winning design ushered in the era of steam locomotion. More recently, the U.S. Federal Communications Commission sponsored a contest to develop the technology for high-definition television.[3] A prize incentive has been proposed for developing new vaccines (see Michael Kremer, 2001) and even for developing the technology for a manned flight to Mars (see Robert Zubrin, 1996). The U.S. Department of Defense (DoD) awards billions of dollars annually to the winners of R&D contests

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[1] The position of the sun was used to determine local time, and the timepiece gave the time in Greenwich, say.

[2] In a voyage across the Atlantic, the metals and lubricating oils inside a timepiece could expand or contract as humidity and temperature changed. This meant that, in principle, a successful trial on a single route might be an aberration.

[3] The prize was a license to operate in the market, so the value differed across firms. Our model encompasses tournaments with different prizes.

(see Fullerton et al., 2002), and a substantial fraction of the basic research done in the United States is funded through grant competitions sponsored by the federal government.

Contests are used to procure goods and services that embody new technologies as well. When a new weapons system is procured, two or more suppliers may perform basic R&D to build a prototype, with only one being selected to produce. The recent procurement of the Joint Strike Fighter by the DoD involved a fly-off competition between prototypes built by the two finalists, with Lockheed-Martin prevailing over Boeing (see Robert Wall and David A. Fulghum, 2001). Elements of research contests are seen in the procurement of a multitude of other goods and services: The construction of new buildings, the publication of books, the development of advertising campaigns, the procurement of realty services, the commissioning of art projects, the procurement of consulting services, and the search for expert witnesses often involve the solicitation of bids from multiple potential suppliers, along with a pilot project or proposal.[4]

In this paper, we study the use of contests to procure an innovation. The procurer might be a private firm requiring a new product or process technology, or a government wishing to encourage basic research or to foster the development of defense technologies, for instance. We focus on a class of contests in which the procurer invites a number of firms to undertake innovative activity and to choose a prize from a (possibly infinite) menu of prizes; the procurer then chooses the innovation that gives the best "value." This class encompasses a wide range of observed contests, as well as bilateral contracts, but it omits some obvious alternatives. For instance, a contest could specify the precise nature of the required innovation (as in the longitude contest). Moreover, for certain innovations, the patent system provides sufficient incentives to generate the desired innovation; the procurer could then license the patented technology. These alternatives may not work well for other innovations, however.

The first reason that the alternatives might not work well is that the quality of the innovation and the effort provided by the innovators may be "unverifiable." Third parties may be unable to observe quality or discern effort through an audit, for instance. This verifiability problem renders unenforceable contracts that are contingent on these variables. Moreover, as the lengthy adjudication process in the longitude contest shows, it may be difficult to enforce a contest (or contract) that specifies the exact nature of the innovation. The problem is especially acute when procuring new technologies, which may be difficult to describe or measure objectively.

Second, the benefits from the innovation may accrue primarily to the procurer rather than the innovator. That is, the investment in R&D may be "cooperative" in the sense of Che and Donald Hausch (1999). This happens when a government procures basic research or a new defense technology with no immediate commercial value. The innovators may lack the expertise to commercialize or license their innovations as well. The patent system would not help then since the innovator would benefit little from an exclusive property right.[5]

Third, the buyer may be unable to charge substantial entry fees to induce a "buy-in" by potential suppliers.[6] The infeasibility of entry fees may result from suppliers' liquidity constraints or from opportunism. In particular, the buyer could collect entry fees but then award the prize to a confederate, or not take delivery at all.

In the face of these problems, contests may perform better than the alternatives because a buyer has an *ex post* incentive to choose the supplier who offered her the greatest net

[4] Advertising agencies typically develop a pilot commercial before signing a contract. Similarly, many homebuyers request showings by multiple brokers before signing exclusive contracts.

[5] The patent system also creates a monopoly distortion, which is likely to be especially harmful when the innovation has a strong "public good" element. See Nancy Gallini and Suzanne Scotchmer (2002) for an excellent survey comparing alternative incentive systems for innovation.

[6] Research contests rarely require substantial entry fees. To the contrary, procurers frequently provide substantial subsidies to the firms selected to participate in research contests. The DoD often expresses concern over the financial stability of suppliers because of a desire to maintain competition.

surplus. This tends to reward the supplier who made the largest investment, which mitigates the verifiability and cooperativeness problems. We study the optimal design of contests, focusing on two issues: (1) how the buyer should design the prize structure, and (2) how she should select the set of participants.

One simple method for determining the winner's prize is to specify it up front. There are many examples of fixed-prize tournaments besides the longitude and steam engine contests.[7] Another possibility is to hold an auction and let the suppliers bid (i.e., demand their prizes). Auctions are used in many procurement settings where *ex ante* investments are important. For example, defense contractors often make R&D investments to produce prototypes and then bid prices for the production contract.[8] Procurement of a high-speed train system in Korea featured a contest in which firms proposed alternative designs (including the French TGV and German ICE) and then bid prices for their systems. Industry publications have touted the benefits of using auctions to make a " 'best value' award decision based on both price and non-price factors" (Richard Rector, 2000).

In between auctions and fixed-prize tournaments are numerous other methods for determining the winner's prize. For instance, the buyer may offer a finite menu of prizes. (Grant competitions sponsored by the National Science Foundation do this, giving applicants options for the duration of the award, travel expenditures, and equipment purchases.) The buyer may also discriminate across firms. We consider a class of mechanisms that includes all of these alternatives.

We show that holding an auction with two firms and no restriction on the set of allowable prizes is optimal for the buyer when the firms have the same investment technology. Each firm's technology is deterministic here, so firms randomize when choosing the quality of their respective innovations, resulting in heterogeneous quality. Nonetheless, a firm with a low-quality innovation can still compete effectively against a firm with a high-quality innovation by asking for a smaller prize. This feature favors auctions relative to contests that limit firms' choice of prizes.

Limiting the number of participants is beneficial because investments are sunk. With large-scale participation, each firm will have a relatively small chance of winning, so the winner's investment (and quality) will tend to be low. This finding is consistent with numerous examples of buyers selecting a small number of finalists (see Taylor, 1995; Fullerton and McAfee, 1999). When firms have different technologies, an auction involving the two most efficient firms is again optimal, with handicapping of the more efficient one through imposition of a maximum allowable prize.

The literature on the design of research contests remains small, but there have been some notable contributions. The aforementioned papers by Taylor (1995) and Fullerton and McAfee (1999) focus on design issues involving the number of participants and the method of restricting entry.[9] Both papers assumed that fixed-prize tournaments are used. Fixed-prize tournaments are simple and effective for a buyer with limited information.[10] First-price auctions outperform fixed-prize tournaments in the current setting, however, and they may require even less information: With *ex ante* symmetric firms, the buyer does not need any information about costs or investments to design an optimal contest.

Fullerton et al. (2002) have independently studied prize structures in contests. Specifically, they compared two methods for determining the winner's prize: first-price auctions and fixed prizes. They found auctions to be superior to a range of fixed prizes when identical innovators follow symmetric strategies. They also presented experimental evidence supporting the desirability of auctions. The current paper con-

[7] See Patrick Windham (1999) for a comprehensive list.

[8] Fred Thompson and L. R. Jones (1994) describe the process in detail. They stress the Packard Commission recommendation to hold competitions between working prototypes instead of "paper competitions" (p. 145). They also note the sharp decrease in the number of "cost-plus" contracts entered into by the DoD, and the corresponding increase in the number of fixed-price contracts (p. 192).

[9] Also related is Benny Moldovanu and Aner Sela (2001), which studies the use of a contest with *multiple* fixed prizes to maximize contestants' *aggregate* effort.

[10] See Fullerton and McAfee (1999, p. 575).

siders a much larger class of contests and design options, including auctions and any possible fixed prize, with no restriction on equilibrium strategies. We also consider asymmetric firms. At the same time, our model considers deterministic innovation technologies whereas Fullerton et al. considered a stochastic technology, which is more realistic. The stochastic technology also introduces an additional benefit from using contests: There may be more experimentation (i.e., multiple innovation "draws"), leading to superior innovations. Thus, the two papers complement each other.

The remainder of the paper is laid out as follows. Section I discusses the model and provides a partial characterization of the equilibria of general contests. We provide a full characterization of two-firm auctions in Section II. In Section III, we identify the optimal contest. Section IV concludes.

I. Model and Preliminary Results

A buyer wishes to procure an innovation from $N \geq 2$ risk-neutral firms. The innovation requires a sunk investment by the firms. Firm i receives an innovation of quality $x \geq 0$ if it invests $\psi_i(x)$. The investment could be a monetary or nonmonetary cost.[11] The functions $\{\psi_i(\cdot)\}_{i=1}^N$ are common knowledge for the firms and the buyer. The firms are unable to realize the value of the innovation directly or through licensing to a third party, so the only way to capitalize on it is to sell to the buyer. The buyer realizes a surplus of $x - p$ from an innovation of quality x, if she pays p. The buyer wishes to procure an innovation from at most one firm.

- *Technology:* The investment cost, $\psi_i(x)$, is strictly increasing and absolutely continuous for $x > 0$, and $\psi_i(0) = 0$ (i.e., zero quality requires no effort). Also, there exists a threshold quality, $\hat{x} > 0$, such that a higher quality is socially unattractive; i.e., $\psi_i(x) > x$ for all $x > \hat{x}$. These assumptions are mild, allowing the function to be nondifferentiable and nonconvex. They also allow for fixed costs, since $\psi_i(x)$ may jump at $x = 0$.

  Throughout, we assume that any two firms are uniformly ranked in terms of efficiency: $\psi_i(x) \leq \psi_j(x)$ if $i < j$. Let $x_i^* \in \arg\max_{x\geq 0}\{x - \psi_i(x)\}$ denote the efficient quality level for firm i, and let $s_i^* := x_i^* - \psi_i(x_i^*)$ be the associated net surplus. Clearly, s_i^* is the maximum surplus firm i can profitably deliver to the buyer.[12] The uniform rankability of cost functions implies $s_i^* \geq s_j^*$ if $i < j$. Consequently, it would be socially optimal for firm 1 to produce an innovation of quality x_1^* (and for no other firm to invest). For the buyer to benefit from holding a contest, at least two firms must be able to deliver positive surplus. Hence, we assume $s_2^* > 0$.
- *Information:* The quality of firm i's innovation, x_i, is common knowledge between the firm and the buyer after it has been chosen, but it is unobservable to other firms. This assumption is sensible when the nature of innovative activity is idiosyncratic, in which case a firm cannot discern the R&D strategies of the other firms or how highly the buyer would value their innovations. As will be seen, if firms randomize in their quality decisions, this moral hazard problem entails adverse selection since the firms' quality choices become their "types."[13] The quality is also unobservable to the courts. Since the innovative effort and quality are unverifiable, one cannot enforce a contract that is contingent on those variables. The identity of the winning firm and its choice of prize are observable and enforceable, however.
- *Contest Mechanism:* The contest mechanism specifies $\mathcal{N} \subset \{1, \dots, N\}$, the set of participants. If the buyer selects $n < N$ firms to participate, we index the firms so that $\mathcal{N} = \{1, \dots, n\}$, with the order again being in terms of efficiency. The mechanism also

[11] Firms sink investments even though they may lose the contest. This feature does not violate the "liquidity constraint" explanation for the infeasibility of entry fees if investments take the form of nonmonetary effort or opportunity costs. Investment costs that are monetary costs are consistent with this explanation when the constraint results from legal restrictions or from opportunism.

[12] That is, s_i^* is the maximum value $x - p$ can take, given that firm i will be paid its cost, meaning $p = \psi_i(x)$.

[13] Faruk Gul (2001) also studies the feature that an agent who randomizes over investments generates an adverse selection problem, in an infinite horizon bargaining model.

specifies a menu of prizes, $\mathcal{P}_i \subset \Re_+$, for each $i \in \mathcal{N}$. Firm i chooses a prize, $p_i \in \mathcal{P}_i$, which it will receive if it wins. Let $\mathcal{P} := \{\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_n\}$. There are no entry fees, and losing firms receive no payments. This means firm i receives a net payoff of $p_i - \psi_i(x_i)$ if it wins with an offer of (x_i, p_i); it receives $-\psi_i(x_i)$ if it loses with that same offer.

The mechanism also specifies how the winning firm is selected. A rule for selecting the winner is a vector, $\mathcal{Q}$, of probability assignment functions, $Q_i : \Re^n_+ \times \Pi_{i \in \mathcal{N}} \mathcal{P}_i \rightarrow [0, 1]$, such that $\Sigma_{i \in \mathcal{N}} Q_i(\mathbf{x}, \mathbf{p}) \leq 1$ for all $(\mathbf{x}, \mathbf{p})$. The functions map a profile of chosen qualities and prizes, $(\mathbf{x}, \mathbf{p})$, into a vector of winning probabilities. Given the unverifiable nature of the innovation, the buyer's only credible choice is the firm that offered her the highest net surplus. Any rule that specified a different choice would not be self-enforcing, and a court could not enforce such a choice since it could not verify whether a breach had occurred.[14] Thus, the probability assignment functions satisfy the selection rule,

$$\text{(R)} \quad Q_i(\mathbf{x}, \mathbf{p}) = 1 \text{ if } x_i - p_i > \max_{j \in \mathcal{N} \setminus \{i\}} x_j - p_j \text{ and } x_i - p_i > 0.$$

Note that (R) above does not pin down a unique allocation since it leaves unspecified what happens when the highest surplus offer is negative and when firms tie at the highest offer. Thus, (R) allows the buyer not to take delivery (if that is beneficial *ex post*) or to commit herself to take delivery, and it permits an arbitrary tie-breaking rule when the buyer is indifferent among offers.

We say that a contest $C := \langle \mathcal{N}, \mathcal{P}, \mathcal{Q} \rangle$ is *feasible* if $\mathcal{Q}$ satisfies the selection rule (R), given $\mathcal{N}$ and $\mathcal{P}$. The set of feasible contests, $\mathcal{C}$, includes a range of contests that are commonly used, as well as bilateral contracts such as fixed-price and buyer-option contracts.[15] It includes first-price auctions ($\mathcal{P}_i = \Re_+$ for all $i \in \mathcal{N}$),[16] and fixed-prize tournaments ($\mathcal{P}_i$ is a singleton). It includes contests in which firms are treated asymmetrically, and ones in which some firms are restricted to a finite menu of prizes. In fact, $\mathcal{C}$ encompasses all mechanisms for which only the winner receives a prize and the prize depends only on the winner's message (e.g., its reported quality).

Timeline: At date 0, the buyer announces a contest, $C = \langle \mathcal{N}, \mathcal{P}, \mathcal{Q} \rangle$. At date 1, each firm $i \in \mathcal{N}$ invests, receives an innovation, and selects a prize from $\mathcal{P}_i$. At date 2, the buyer observes $(\mathbf{x}, \mathbf{p})$ and takes delivery from at most one firm. The winning firm delivers its innovation and collects the prize it selected at date 2. Our model is formally identical to one in which each firm chooses its quality first and then its prize, or vice versa, as long as other firms' first-round actions are unobservable when the second action is taken.[17]

Given a contest $C \in \mathcal{C}$, a (pure) strategy for firm $i \in \mathcal{N}$ consists of the choice of a quality, $x_i \geq 0$, and a prize, $p_i \in \mathcal{P}_i$. If the firms in $\mathcal{N}$

[14] A selection rule that is completely independent of $(\mathbf{x}, \mathbf{p})$, such as random selection or always selecting a particular firm, is feasible. These rules generate no surplus for the buyer, however, so we ignore these possibilities. One can imagine a more sophisticated rule under which the buyer commits to a set of probabilities of awarding the prize based on her preferences over the firms' quality and prize choices. Committing to a complex random selection rule may be difficult from a practical standpoint. The analysis required for this general model also departs from the method used here. We thank a referee for raising this possibility.

[15] This set does not include mechanisms that require the buyer and winning firm to report the latter's quality choice, for example. Since the quality is common knowledge between the two parties, one can induce truthful reporting for free, so the first-best is attainable. Such mechanisms are unobserved in practice, possibly because such messages are susceptible to manipulation.

[16] At first glance, our selection rule may appear different from the conventional first-price auction since the firm offering the ***highest surplus*** is selected here, rather than the one offering the *lowest price*. The selection rule is natural in the procurement setting in which firms offer different quality levels. In particular, it coincides with the standard rule in which the lowest bidder wins, when the offered qualities are the same.

[17] We have assumed that quality is private information at the time firms bid their prizes. Suppose the buyer could credibly reveal the quality choices before the firms bid. Revealing the quality choices always admits a bad equilibrium (for the buyer) in which the most efficient firm successfully preempts the other firms and offers zero surplus.

choose $(\mathbf{x}, \mathbf{p}) \in \Re^n_+ \times \Pi_{i\in\mathcal{N}} \mathcal{P}_i$, firm i receives an expected payoff $u_i := p_i Q_i(\mathbf{x}, \mathbf{p}) - \psi_i(x_i)$. We will focus on Nash equilibria in the mixed extension of the strategies.

Consider an equilibrium of an arbitrary contest. It will be convenient to focus directly on the net surplus that firm $i \in \mathcal{N}$ offers in equilibrium, $s_i := x_i - p_i$. Let $G_i : \Re \to [0, 1]$ denote the cumulative distribution function (c.d.f.) of firm i's net surplus offer in that equilibrium. Let S_i denote the support of s_i. Formally, $s_i \in S_i$ if $G_i(s_i + \varepsilon) - G_i(s_i - \varepsilon) > 0$ for any $\varepsilon > 0$. Denote the supremum and infimum of S_i by $\bar{s}_i$ and $\underline{s}_i$, respectively.[18] The aggregate support is $S := \cup_{j\in\mathcal{N}} S_j$, with $\bar{s}$ and $\underline{s}$ denoting its supremum and infimum, respectively.

We now provide a partial characterization of equilibrium for a given contest $C \in \mathcal{C}$. The first result shows that the procurer does not benefit from inviting a single firm to participate (i.e., using a bilateral contract).

LEMMA 1: *In any equilibrium of $C \in \mathcal{C}$ with $|\mathcal{N}| = 1$, the buyer receives nonpositive net surplus with probability 1.*

PROOF:

Suppose, to the contrary, that there exists an equilibrium with $S \cap \Re_{++} \neq \emptyset$. Fix any $s \in S \cap \Re_{++}$ and an associated choice, $(x, p) \in \Re_+ \times \mathcal{P}_1$, with $s = x - p > 0$. If the firm offers (x', p) instead, with $p < x' < x$, the buyer will still take delivery. Since the firm receives the same prize, but lowers its investment, the expected payoff rises, which yields a contradiction.[19]

Remark 1: This lemma, like the rest of the paper, assumes away any renegotiation possibility. In practice, if the buyer retains the right to refuse delivery, she may exercise that right so as to trigger renegotiation if the lone firm would capture any surplus if the buyer accepted delivery. Such a renegotiation possibility does not alter the conclusion of Lemma 1, however. Che and Hausch (1999) show that the firm will be severely held up when the buyer has all the bargaining power (as is the case here), leaving the firm with no incentive for investment.

Suppose, henceforth, that $\mathcal{N}$ contains at least two firms. We now present a series of lemmas that are useful for characterizing equilibria of general contests.

LEMMA 2: *$S \cap \Re_{++}$ has no mass points.*

LEMMA 3: *Suppose $\bar{s} > 0$. Then, $S \cap \Re_+ = [\underline{s}, \bar{s}] \cap \Re_+$.*

LEMMA 4: *For every open interval $I \subset S \cap \Re_{++}$, at least two firms have* $\Pr\{s_i \in I\} > 0$.

LEMMA 5: *Each firm i has an infimum surplus $\underline{s}_i \leq 0$, and a supremum $\bar{s}_i \leq s^*_2$.*

Lemma 2 shows that no strictly positive value of surplus is offered with positive probability. Hence, any equilibrium the buyer would wish to implement is in mixed strategies.[20] Moreover, $G_i(s)$ must be continuous for $s > 0$. Next, Lemma 3 shows that there cannot be a gap in the set of positive realizations of S. Lemma 4 says that at least two firms offer surplus in any nonnegative interval in S. Finally, Lemma 5 shows that the infimum surplus offered by any

[18] As will be seen, the supremum is always attained, and the infimum is attained in all cases of practical interest. Hence, there is little loss in interpreting them as the maximum and minimum, respectively.

[19] While this argument appears obvious, the result requires all three conditions of the model. If the level of investment or the innovation quality were verifiable, an enforceable contract could be written that specified the first-best level of investment. With entry fees, the buyer could set $\mathcal{P}_1 = \{x^*_1\}$, with no commitment to take delivery [so $Q_1(x', p') = 0$ for $x' - p' < 0$], and charge an entry fee equal to s^*_1. This would induce the firm to choose quality x^*_1 and receive a payoff of $x^*_1 - s^*_1 - \psi_1(x^*_1) = 0$, giving the buyer a net surplus of s^*_1. Likewise, if the investment were not cooperative (e.g., it just reduced the firm's cost of production), the first-best outcome could be attained by a simple fixed-price contract.

[20] To see why a pure-strategy equilibrium cannot arise when $n \geq 2$, suppose that firm i offers $s > 0$ with probability 1. Doing so requires a sunk investment, so the firm must win with positive probability. Also, no other firm must be offering a surplus equal to s, or slightly lower, since that would be dominated by offering slightly more than s (which would yield a discrete jump in the probability of winning, with only a slightly higher investment cost). In that case, firm i would be strictly better off lowering its surplus offer. Hence, there cannot exist a pure-strategy equilibrium.

firm cannot be strictly positive, and the supremum cannot exceed the maximum surplus firm 2 could profitably offer.

Since investments are sunk, it is natural that these properties mirror those found in the equilibria of all-pay auctions (see Michael Baye et al., 1996).[21] At the same time, the model here differs since the cost function is nonlinear and the value of the prize is endogenous. The structure of the proofs is essentially the same, however, so the proofs are omitted (they are available upon request).

We next present a result that is critical for characterizing equilibria of a given contest and for comparing equilibria from different contests. A firm will offer any equilibrium surplus in the most efficient way. That is, firm i will choose $(x_i, p_i) \in \Re_+ \times \mathcal{P}_i$ to maximize its expected payoff from offering s. This implies

$$(1) \quad u_i = \sup_{x \geq 0, p \in \mathcal{P}_i} \left\{ p \prod_{j \in \mathcal{N} \setminus \{i\}} G_j(s) - \psi_i(x) : \text{s.t. } x - p = s \right\},$$

for any $s \in S_i \cap \Re_{++}$. [Given Lemma 2, firm i wins with probability $\Pi_{j \in \mathcal{N} \setminus \{i\}} G_j(s)$ when offering s.] This condition is equivalently characterized by its dual:

$$(2) \quad \prod_{j \in \mathcal{N} \setminus \{i\}} G_j(s) = \inf_{x \geq 0, p \in \mathcal{P}_i} \left\{ \frac{u_i + \psi_i(x)}{p} : \text{s.t. } x - p = s \right\}.$$

We now give the result.

LEMMA 6: *For (almost) every $s \in S_i \cap \Re_{++}$, $G_j(s)$ must satisfy (2) for all $j \neq i$. If the infimum is attainable, firm i chooses a minimizer of (2) when offering such an s.*

Equation (2) simplifies the process of finding an equilibrium in an arbitrary contest. Once we identify the equilibrium payoffs and the support of the net surplus offered in equilibrium, (2) helps uncover the equilibrium distribution function, G_j. In particular, it pins down G_j exactly if $|\mathcal{N}| = 2$. We now turn our attention to auction contests with two firms.

II. Equilibrium of Auctions with Two Firms

In this section, we present a complete characterization of equilibrium for first-price auctions with two firms.[22] Auctions merit special attention because of their frequent use and because of their prominence in the theoretical literature. First-price auctions in particular are used in many procurement settings. Moreover, we will subsequently show that the optimal contest is a two-firm auction.

Let $\mathcal{N} = \{i, j\}$, and let $\mathcal{P}_k = [0, \bar{p}_k]$ for $k = i, j$, where $\bar{p}_k$ represents firm k's maximum allowable prize. With just two firms participating, Lemmas 3, 4, and 5 imply that each firm must offer surplus everywhere in $(0, \bar{s}]$ in equilibrium. Lemma 6 then pins down the c.d.f. of each firm's surplus offering for $s \in (0, \bar{s}]$, once $\bar{s}$ and u_k are known. To characterize the latter, let

$$s_k^*(\bar{p}_k) := \max_{x \in \Re_+, p \in [0, \bar{p}_k]} \{x - p : \text{s.t. } p \geq \psi_k(x)\}$$

denote the highest surplus firm k can profitably offer, $k = i, j$. We restrict attention to values of $\bar{p}_k$ such that $s_k^*(\bar{p}_k) > 0$ for $k = i, j$; otherwise, no firm would offer a strictly positive surplus in equilibrium.

Suppose $s_i^*(\bar{p}_i) \geq s_j^*(\bar{p}_j)$. (There is no presumption that $i < j$ here, so firm i need not be more efficient.) If firm i were to offer surplus $s_j^*(\bar{p}_j)$, it would win with probability 1 and earn

[21] Similar features are also found in oligopoly models with price dispersion (see Hal Varian, 1980, or McAfee, 1994, for example).

[22] Auctions with two firms have unique equilibrium payoffs here. A full characterization of equilibria with $|\mathcal{N}| \geq 3$ is cumbersome due to the multiplicity of equilibria. It is also unnecessary since the optimal design analysis only requires the partial characterization reported in the previous section.

$$u_i^*(\bar{p}_i) := \max_{p \in [0,\bar{p}_i]} p - \psi_i(p + s_j^*(\bar{p}_j)),$$

since firm j cannot profitably offer more than $s_j^*(\bar{p}_j)$, and it never puts mass at $s = s_j^*(\bar{p}_j) > 0$. Clearly, firm i will earn at least this much in any equilibrium. We establish below that firm i actually receives this payoff in equilibrium, while firm j earns a payoff of zero.

For ease of presentation, suppose the buyer commits to take delivery even when the higher surplus offer is negative, and she favors firm i when the two firms tie at zero surplus:

$$(\mathrm{R}') \quad Q_k(\mathbf{x}, \mathbf{p}) = 1 \text{ if } x_k - p_k > x_l - p_l \text{ for } k, l = i, j;$$

$$\text{and } Q_i(\mathbf{x}, \mathbf{p}) = 1 \text{ if } x_i - p_i = x_j - p_j = 0.$$

This selection rule satisfies (R), but it imposes additional structure. The tie-breaking rule is needed only to ensure existence of an equilibrium (just as in a Bertrand game with homogeneous products but heterogeneous unit costs). It will become clear that the restriction to (R′) entails no loss to the buyer, and any subsequent reference to a two-firm auction will implicitly invoke (R′).[23]

The following proposition characterizes equilibrium for two-firm auctions, and it demonstrates existence.

PROPOSITION 1: *Consider an auction with $\mathcal{N} = \{1, 2\}$ in which firms 1 and 2 face maximum allowable prizes of $\bar{p}_1$ and $\bar{p}_2$, respectively. Suppose $s_i^*(\bar{p}_i) \geq s_j^*(\bar{p}_j) > 0$ for $i, j \in \mathcal{N}$, $i \neq j$. Given* (R′), *an equilibrium exists. In any equilibrium, $S_i = [0, s_j^*(\bar{p}_j)]$, $S_j \cap \Re_+ = [0, s_j^*(\bar{p}_j)]$, and firms i and j receive expected payoffs of $u_i^*(\bar{p}_i)$ and* 0, *respectively. Firms i and j offer surplus in* $(0, s_j^*(\bar{p}_j)]$ *according to the c.d.f.s,*

$$G_i(s) = \min_{p \in [0,\bar{p}_j]} \frac{\psi_j(p + s)}{p},$$

and

$$G_j(s) = \min_{p \in [0,\bar{p}_i]} \frac{u_i^*(\bar{p}_i) + \psi_i(p + s)}{p},$$

respectively.

The selection rule (R′) ensures that firm i never offers negative surplus (i.e., $\underline{s}_i = 0$). Hence, the surplus accruing to the buyer cannot be negative. Since the equilibrium characterization is the same for $s > 0$ for any selection rule satisfying (R), the buyer can never be worse off using (R′).[24]

An important case arises when the firms face no maximum prize restriction (i.e., $\bar{p}_1 = \bar{p}_2 = \infty$). We focus on this case for the remainder of this section. The equilibrium characterization for this case is obtained as a simple application of Proposition 1.

COROLLARY 1: *Assume $\mathcal{N} = \{1, 2\}$ and $\mathcal{P}_i = \Re_+$ for $i = 1, 2$. In any equilibrium, $S_1 = [0, s_2^*]$ and $S_2 \cap \Re_+ = [0, s_2^*]$. Firms 1 and 2 earn expected payoffs of $s_1^* - s_2^*$ and 0, respectively. The firms offer surplus over $(0, s_2^*]$ according to the c.d.f.s:*

$$G_1^a(s) := \min_{p \in \Re_+} \frac{\psi_2(p + s)}{p}, \tag{3}$$

and

$$G_2^a(s) := \min_{p \in \Re_+} \frac{s_1^* - s_2^* + \psi_1(p + s)}{p}. \tag{4}$$

PROOF:

Proposition 1 applies since a sufficiently high maximum allowable prize has no effect on behavior. The characterization and existence

[23] The equilibrium outcome for the buyer will be precisely the same if she retains the option to reject delivery and exercises it whenever the highest offer is strictly negative. See Che and Gale (2000).

[24] If the buyer retains the option not to take delivery, the equilibrium characterization is the same except that both firms may offer negative surplus. Such offers will be rejected, however. The c.d.f.s for positive surplus are the same for any rule satisfying (R), so the surplus accruing to the buyer will be the same as with (R′).

follow by noting that $s_k^*(\infty) = s_k^*$ and $u_1^*(\infty) = s_1^* - s_2^*$.

To get some intuition for the equilibrium expected payoffs here, note that firm 1 could choose the socially optimal quality, x_1^*, and offer a surplus of s_2^* by bidding $x_1^* - s_2^*$. Winning would give the firm a payoff of $x_1^* - s_2^* - \psi_1(x_1^*) = s_1^* - s_2^*$. Firm 2 cannot beat an offer of s_2^*, so its best response would be to offer zero surplus. But firm 1 would then have an incentive to lower its surplus offer. It follows that the equilibrium must be in mixed strategies (as shown in Lemma 2), and firm 1 must receive at least $s_1^* - s_2^*$. Firm 2 must receive at least zero. In fact, these lower bounds are both attained.

This corollary does not describe firms' equilibrium quality and prize choices, but they can easily be derived from the surplus offer strategies. Invoking duality, Lemma 6 shows that, when offering a net surplus $s \in (0, s_2^*]$, firm $i = 1, 2$ chooses quality

$$x_i^a(s) \in \arg\min_{x \in \Re_+} \frac{u_i^a + \psi_i(x)}{x - s}, \tag{5}$$

where $u_1^a = s_1^* - s_2^*$ and $u_2^a = 0$. The distribution of firm i's quality choice (or "type") can then be derived from G_i^a:

$$\begin{aligned} F_i^a(x) &= \Pr\{x_i^a(s) \leq x\} \\ &= G_i^a(\sup\{s : x_i^a(s) \leq x\}), \end{aligned} \tag{6}$$

where the second equality holds since $x_i^a(\cdot)$ is nondecreasing.[25]

The remainder of this section characterizes the equilibrium of a two-firm auction further. (Readers interested in optimal contest design may skip this part and go directly to the next section.) Of particular interest is how the cost structure affects the endogenous distribution of quality. It is commonly assumed in the auction literature that each bidder's type has an atomless distribution over an interval. The next proposition identifies sufficient conditions for the equilibrium distributions to display this property. It refers to the following differentiability condition:

Condition D: For each $i \in \mathcal{N}$, $\psi_i(x)$ is differentiable and $\psi_i'(0) = 0$.

PROPOSITION 2: (*i*) *Given Condition D, each firm's equilibrium quality choice, x, follows an atomless distribution for $x > 0$, and firm 1 bids the prize*

$$b_1(x) := \frac{s_1^* - s_2^* + \psi_1(x)}{\psi_1'(x)} \textit{ for } x \in X_1^a, \textit{ where}$$

$$X_1^a := \{z : x_1^a(s) = z \textit{ for some } s \in (0, s_2^*]\},$$

while firm 2 bids the prize

$$b_2(x) := \frac{\psi_2(x)}{\psi_2'(x)} \textit{ for } x \in X_2^a, \textit{ where}$$

$$X_2^a := \{z : x_2^a(s) = z \textit{ for some } s \in (0, s_2^*]\}.$$

(*ii*) *If $\psi_i(\cdot)$ is strictly convex for $i = 1, 2$, the support of each firm's quality choice forms an interval.*

The proposition also shows that, once a firm has chosen its quality according to an atomless distribution, it chooses a prize deterministically, which is consistent with the equilibrium of standard auctions.[26] The following example illustrates how quadratic cost functions generate a particularly simple case—a uniform distribution over an interval.

Example 1 (*A quadratic cost function*): Suppose the buyer selects two firms, each of which has the cost function $\psi(x) = \frac{1}{2}x^2$. In the unique equilibrium of the first-price auction, the two firms

[25] To see this, consider $\phi(x; s) := \dfrac{x - s}{u_i^a + \psi_i(x)}$ for some $u_i^a \geq 0$, which is the reciprocal of the function that is minimized in (3) and (4). Hence, $x_i^a(s)$ coincides with some selection of arg $\max_{x \geq s} \phi(x; s)$. Now observe that, for $x \geq s$, $\phi(x; s)$ satisfies the *strict single-crossing property* in $(x; s)$, as defined by Paul Milgrom and Chris Shannon (1994). Their Monotone Selection Theorem (Theorem 4') then implies that $x_i^a(s)$ is nondecreasing.

[26] The one difference is that firms may put positive mass at zero surplus here. Interpreting $(x, p) = (0, 0)$ as nonparticipation, the auction with Condition D coincides with a standard auction with random participation.

choose quality uniformly over [0, 1]. That is, a quadratic cost function generates uniformly distributed types. Each firm asks for a prize equal to one-half of its chosen quality, so it offers net surplus distributed uniformly over [0, ½].

If Condition D does not hold, the set of equilibrium types could be discrete. Lemma 2 then implies that firms must randomize over prizes, after choosing their respective qualities. The following example illustrates this possibility.

Example 2 (A nondifferentiable cost function): Suppose the buyer selects two firms to participate, each of which has the cost function

$$\psi(x) = \begin{cases} \frac{1}{2}x & \text{for } x \in [0, 1], \\ 2x - \frac{3}{2} & \text{for } x > 1. \end{cases}$$

The cost function is not differentiable at $x = 1$, and $x_1^* = x_2^* = 1$ as a consequence. In equilibrium, each firm randomizes between $x = 0$ and $x = 1$ with equal probability. Each firm bids a zero prize if $x = 0$, and bids randomly in [½, 1] according to

$$K(b) := \begin{cases} \dfrac{2b - 1}{2b} & \text{for } b \in \left[\dfrac{1}{2}, 1\right), \\ 1 & \text{for } b \geq 1, \end{cases}$$

if $x = 1$. Thus, each firm puts probability mass of ½ at $s = 0$ and offers $s \in (0, ½]$ according to the c.d.f. $G^a(s) = \min\left\{\dfrac{1}{2(1 - s)}, 1\right\}$.

Next, we explore how the contract is allocated, given the firms' quality choices. Part of the result invokes the following *increasing differences* condition on costs:

Condition ID: For $x' > x$, $\psi_1(x') - \psi_1(x) < \psi_2(x') - \psi_2(x)$.

Given differentiability, this condition reduces to the more efficient firm having a lower marginal cost than the less efficient one does.

PROPOSITION 3: *(i) If $\psi_1(\cdot) = \psi_2(\cdot)$, the firm that chooses the higher x wins with probability 1 in equilibrium. (ii) Given Condition ID, the more efficient firm asks for a (weakly) higher prize than the less efficient one would, given the same strictly positive quality. If Condition D also holds, the more efficient firm asks for a strictly higher prize than the less efficient one would, given the same strictly positive quality.*

The proposition says that symmetry yields an efficient allocation in equilibrium, given the quality choices. *Ex post* efficiency is not guaranteed with asymmetric firms, however. A less efficient firm may win, despite offering lower quality, because a more efficient firm will demand a larger prize, all else equal. A similar distortion arises in an asymmetric auction with exogenous distributions (see Eric Maskin and John Riley, 2000).

While Proposition 3 describes how a firm bids, conditional on its quality, it does not characterize overall net surplus offerings. To that end, a thought experiment is useful. Consider two firms with $\psi_1(x) < \psi_2(x)$ for all $x > 0$. Now suppose we replace one of the firms with another that is exactly as efficient as the remaining firm. We can then compare the outcome of an auction in which firm i has the cost function ψ_i, $i = 1, 2$, with an auction in which both firms have ψ_1 or both have ψ_2. Let $\hat{G}_i^a$ denote the c.d.f. for surplus in a first-price auction when both firms have ψ_i, $i = 1, 2$. $G_i^a(\cdot)$ again denotes the c.d.f. in the original asymmetric auction contest for $i = 1, 2$, as given in (3) and (4).

PROPOSITION 4: *Given Condition ID,*

$$\hat{G}_1^a(s) < G_1^a(s) = \hat{G}_2^a(s) \leq G_2^a(s),$$

for $s \in (0, s_2^*)$. If Condition D also holds, the last inequality holds strictly.

This result reveals several interesting features of asymmetric auctions. First, $G_1^a(s) \leq G_2^a(s)$ means the more efficient firm tends to offer higher net surplus than the less efficient one does. That is, although the more efficient firm demands a larger prize when offering a given quality [see Proposition 3(*ii*)], it tends to offer higher quality, which more than compensates for the larger prize. Hence, in terms of surplus offerings, the more efficient firm *competes more aggressively* (i.e., offers higher surplus, on average) than the less efficient firm does. Second, $G_1^a(s) > \hat{G}_1^a(s)$ means that a firm competes less aggressively when facing a less efficient firm than when facing an equally efficient firm. At the same time, $G_2^a(s) \geq \hat{G}_2^a(s)$ means that a firm competes less aggressively when facing a more efficient firm than an equally efficient firm. When the firms become asymmetric, the less efficient firm becomes pessimistic about its chances, so it competes less aggressively. Roughly speaking, this allows the more efficient firm to compete less aggressively as well. Such a *preemption effect* also arises in all-pay auctions (see Baye et al., 1993, for example), but not necessarily in a standard first-price auction with independent private values.[27]

The preemption effect has one further implication: If the buyer selects two firms for an auction contest, she may not select the most efficient ones. Instead, she may invite firms that are less efficient but more evenly matched. For instance, suppose there are three potential suppliers, with $\psi_1(\cdot) < \psi_2(\cdot) = \psi_3(\cdot)$. Proposition 4 implies that if two firms are selected to participate, $G_1^a(s)G_2^a(s) > G_2^a(s)G_3^a(s)$. This inequality means that the winning surplus when firms 2 and 3 participate stochastically dominates the winning surplus when firms 1 and 2 do. Since the buyer obtains the higher surplus offered by the two firms, more surplus accrues to the buyer (in the sense of stochastic dominance) when firms 2 and 3 participate rather than firms 1 and 2. The difference in efficiency reduces competition, causing both firms to offer lower surplus, on average, than if firm 1 were replaced by another firm with the same technology as firm 2. This result will hold even when the two less efficient firms have different technologies, as long as they are sufficiently similar.

Example 3 (*Asymmetric auction contest*): Suppose the buyer selects two firms with costs

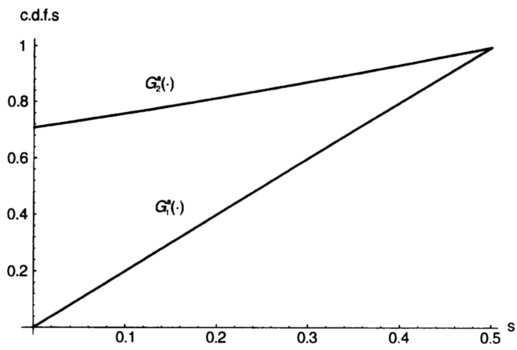

FIGURE 1. SURPLUS OFFERING UNDER AN ASYMMETRIC AUCTION CONTEST

$\psi_1(x) = \frac{1}{4}x^2$ and $\psi_2(x) = \frac{1}{2}x^2$. These firms offer surplus in [0, ½] according to c.d.f.s,

$$G_1^a(s) = 2s \text{ and } G_2^a(s) = \frac{s + \sqrt{2 + s^2}}{2}.$$

As is shown in Figure 1, $G_2^a(s) > G_1^a(s)$ for $s < \frac{1}{2}$. Since $\hat{G}_2^a(\cdot) = G_1^a(\cdot)$, firm 2 would tend to offer greater surplus when facing another firm with ψ_2 than when facing firm 1. This means the buyer would benefit from selecting two firms with ψ_2 rather than firms 1 and 2.

III. Optimal Contest Design

We have so far provided a set of necessary conditions for contest equilibria, and we have characterized equilibria for first-price auctions with two firms. We now search for an optimal contest. The analysis is complicated by the endogeneity of investments and the resulting use of mixed strategies.[28] The duality property developed earlier plays a crucial role here.

A. *Symmetric Firms*

Consider the case in which the two most efficient firms are equally efficient. We now show that the optimal contest is a first-price auction with only firms 1 and 2 as participants.

[27] In an asymmetric first-price auction, an efficient (i.e., high value) bidder competes less aggressively when it faces a less efficient bidder, but an inefficient (i.e., low value) bidder competes more aggressively when it faces a more efficient bidder. See Maskin and Riley (2000).

[28] For instance, the equilibrium allocation rule does not directly pin down the buyer's welfare (as would be the case if revenue equivalence held), since x_i is endogenous.

PROPOSITION 5: *Suppose* $\psi_1(\cdot) = \psi_2(\cdot) =: \psi(\cdot) \leq \psi_i(\cdot)$ *for* $i \geq 3$. *A first-price auction satisfying* (R′) *in which only firms 1 and 2 participate yields the buyer a* (*weakly*) *higher expected surplus than does any other contest in* $\mathcal{C}$.

PROOF:

Consider a first-price auction involving firms 1 and 2. Since the firms are equally efficient, Corollary 1 implies that both earn an expected payoff of zero. Corollary 1 also implies that firm 1's equilibrium support is $S_1^a = [0, s_2^*]$, and the firms adopt a common c.d.f.,

$$(7) \qquad G^a(s) := \min_{p \in \Re_+} \frac{\psi(s+p)}{p},$$

for any $s \in (0, s_2^*]$.

We now show that the surplus accruing to the buyer in an arbitrary contest is (weakly) dominated by that in a first-price auction involving firms 1 and 2. To that end, fix an arbitrary contest, $C = \langle \mathcal{N}, \mathcal{P}, \mathcal{Q} \rangle$ in $\mathcal{C}$, and an associated equilibrium. Let S be the support of the equilibrium surplus, let $G_i : S \to [0, 1]$ be the c.d.f. of the surplus offered by firm $i \in \mathcal{N}$, and let $u_i \geq 0$ be firm i's equilibrium expected payoff. By Lemma 5, $S \cap \Re_{++} \subseteq [0, s_2^*]$.

Fix any $s \in S \cap \Re_{++}$.[29] Lemmas 2, 4, and 5 imply that $G_i(s) > 0$, $G_i(s)$ is continuous at s for all $i \in \mathcal{N}$, and at least two firms have positive density in an open interval I that contains s, given $s \in (0, \bar{s})$. Suppose firms k and l have positive density in I. Then,

$$(8) \qquad \prod_{j \in \mathcal{N} \setminus \{k\}} G_j(s) = \inf_{p_k \in \mathcal{P}_k} \frac{u_k + \psi_k(s + p_k)}{p_k} \geq \min_{p_i \in \Re_+} \frac{\psi(s+p_i)}{p_i} = G^a(s),$$

where the first equality follows from Lemma 6, the inequality holds since $u_k \geq 0$ and $\mathcal{P}_i \subset \Re_+$, and the last equality follows from (7). By the same argument,

$$(9) \qquad \prod_{j \in \mathcal{N} \setminus \{l\}} G_j(s) \geq G^a(s).$$

Multiplying (8) and (9) on both sides, we get

$$\left(\prod_{j \in \mathcal{N}} G_j(s)\right)\left(\prod_{j \in \mathcal{N} \setminus \{k,l\}} G_j(s)\right) \geq G^a(s)^2.$$

Since $G_j(s) \in (0, 1]$, it follows that

$$\prod_{j \in \mathcal{N}} G_j(s) \geq G^a(s)^2.$$

The left-hand side is the c.d.f. of the first order statistic of surplus in the arbitrary contest; the right-hand side gives the corresponding expression for a first-price auction involving firms 1 and 2. Since the latter is smaller for any $s \in S \cap \Re_{++}$, and since the buyer collects the highest surplus offered in a given contest, the auction generates greater surplus for the buyer in the sense of first-order stochastic dominance.

Remark 2: The surplus ranking is strict if the inequality in (8) or (9) is strict. This holds if $u_j > 0$ for some firm j, if $\mathcal{P}_j$ is binding (see Example 4 below), or if more than two firms are active in an interval of s (see Example 5 below).

Auctions dominate all other contests in the class, be they symmetric or asymmetric, for a large class of cost functions that includes nondifferentiable and nonconvex ones.[30] An auction is desirable because it gives firms two means of competing. In a fixed-prize tournament, a firm with a low-quality innovation

[29] We focus exclusively on $s > 0$ since the buyer gets $s \geq 0$ in equilibrium in an auction, and $s < 0$ can only make her worse off, *ex post*, in an arbitrary contest.

[30] We have assumed that entry fees are infeasible, but the above result casts doubt on the value of entry fees when the buyer commits to take delivery. Entry fees seem most viable when the buyer must take delivery since the option not to take delivery can be used opportunistically to collect entry fees but not award a prize. If the buyer commits to take delivery in the two-firm auction, however, firms' rents are completely dissipated, even without entry fees. Charging entry fees can only cause nonparticipation by a firm, which will result in a zero (or possibly negative) surplus for the buyer.

poses no threat to a firm with a high-quality innovation. In an auction, by contrast, the former firm can still compete effectively by asking for a smaller prize. This added competition causes firms to offer higher surplus, on average, in an auction than in other contests. The next example highlights this point.

Example 4 (*Auctions vs. tournaments*): Consider a tournament with two firms, each of which has $\psi(x) = \frac{1}{2}x^2$. In a tournament with a fixed prize, P, a firm with quality x receives an expected payoff of the form $PF^t(x) - \psi(x)$, where F^t denotes the c.d.f. for quality. Since the firms receive zero expected payoff in equilibrium, they each choose quality randomly from the interval $[0, \sqrt{2P}]$ according to $F^t(x) = \psi(x)/P = x^2/2P$. The optimal prize in the tournament is then $P = \frac{8}{25}$ and the buyer's expected net surplus is $\frac{8}{25}$. The corresponding auction equilibrium is given in Example 1. Since both firms offer surplus uniformly over $[0, \frac{1}{2}]$, the buyer's expected net surplus in the auction is $\frac{1}{3} > \frac{8}{25}$.

This example also illustrates why revenue equivalence breaks down. Auctions and tournaments both select the firm offering the highest quality, but they induce different investment decisions, so the amount of surplus created differs.[31]

The optimality of first-price auctions here is reminiscent of the results in Roger Myerson (1981) and John Riley and William Samuelson (1981) that demonstrate the optimality of standard auctions (with a reserve price). An important difference is that our result arises with *ex ante* investment decisions, meaning that bidders' types are endogenous. This difference has important consequences for the precise form of the optimal mechanism: there is not a binding reserve here, and only two firms are invited to participate. The following example illustrates this latter point.

Example 5 (*The effect of increasing the number of firms*): Suppose the buyer invites $n \geq 2$ firms with $\psi(x) = \frac{1}{2}x^2$ to participate in an auction. In the symmetric equilibrium, the surplus accruing to the buyer (i.e., the first order statistic of the surplus offers) is distributed over $[0, \frac{1}{2}]$ according to the c.d.f. $G^a(s; n)^n = \min\{(2s)^{n/(n-1)}, 1\}$. It is immediate that $G^a(\cdot; n)^n$ stochastically dominates $G^a(\cdot; m)^m$ for $n < m$, so it does not pay to increase the number of participants.

In the symmetric, independent-private-values case, increasing the number of bidders is always desirable since it can only increase competition. That conclusion does not follow when types are endogenously distributed. As more firms participate, each firm becomes increasingly reluctant to make a sunk investment since it becomes more pessimistic about its chances of winning. While competition is necessary to motivate investment, too much competition is harmful since the losing firms' investments are wasted. For this reason, the buyer does not benefit from having more than two firms participate.[32] An asymmetric contest that induces one firm to win with a very high probability is undesirable as well. Such an asymmetric contest would make the favored firm too passive (by demanding too much or investing too little).

[31] Our class of contests does not include second-price auctions. Suppose the firm that offers the highest net surplus wins and is required to match the second-highest net surplus, as in Che (1993). Such a format does not work here since quality is unverifiable. Now consider a variant of the second-price auction in which the firm bidding the lowest prize wins and receives the second lowest prize (and let the buyer break ties any way she likes). Such a format does not work well either since firms have a weakly dominant strategy of bidding a zero prize, which would give them no incentive to invest in equilibrium. With a minimum allowable prize of P, say, all active firms bid P, so the second-price auction is then isomorphic to a tournament with a prize of P (since the buyer will prefer the firm with the highest quality). Finally, the variant in which the winning firm receives the highest losing prize bid performs (weakly) worse than the first-price auction.

[32] A similar result and insight can be found in auctions with entry (Dan Levin and James Smith, 1994, for example) and for research tournaments (Taylor, 1995; Fullerton and McAfee, 1999). The result that two is the optimal number of contestants depends somewhat on the deterministic technology in our model. If firms' innovation qualities are uncertain, there are social gains from having additional firms making innovation "draws," especially when individual firms are constrained in how many draws they can take.

B. *Asymmetric Firms*

We have shown the optimality of auctions when firms are *ex ante* symmetric. Since firms may differ in their ability to perform R&D, we now consider asymmetric cost functions. In particular, we consider the possibility that $\psi_1(x) < \psi_2(x)$ for an open interval in $[0, x_1^*]$. The duality argument above would still favor simple first-price auctions if not for the rents that accrue to the most efficient firm. This latter feature can favor other contests that yield lower rents. In particular, an auction that handicaps the most efficient firm may now be desirable.

Consider a two-firm auction in which firm 1 faces a maximum allowable prize of $\bar{p}_1 = \bar{p}$, while firm 2 faces no restriction (i.e., $\bar{p}_2 = \infty$). Proposition 1 shows that, if $s_1^*(\bar{p}) \geq s_2^*(\infty) = s_2^*$, firm 1 receives an expected payoff of

$$u_1^*(\bar{p}) = \max_{p \in [0,\bar{p}]} p - \psi_1(p + s_2^*)$$

in equilibrium (and firm 2 receives zero). Note that $u_1^*(\cdot)$ is continuous and nondecreasing. In particular, it attains a maximum of $s_1^* - s_2^*$ at $\bar{p} = x_1^* - s_2^*$, and it remains constant as $\bar{p}$ rises thereafter.

Let $\bar{p}^*$ be the smallest value of $\bar{p}$ such that $u_1^*(\bar{p}) = 0$. If firm 1's maximum allowable prize is $\bar{p}^*$, neither firm earns any rents. By Proposition 1, firms 1 and 2 then offer surplus over $(0, s_2^*]$ according to the c.d.f.s,

$$\bar{G}_1^a(s) := \min_{p \in \Re_+} \frac{\psi_2(p + s)}{p} \tag{10}$$

and

$$\bar{G}_2^a(s) := \min_{p \in [0,\bar{p}^*]} \frac{\psi_1(p + s)}{p}, \tag{11}$$

respectively, in equilibrium. Comparing (10) and (11) with (3) and (4) reveals that, while the maximum prize restriction does not affect firm 1's surplus offering, it does raise firm 2's surplus offering in the sense of stochastic dominance. Intuitively, the restriction attenuates firm 1's efficiency advantage, which makes firm 2 compete more aggressively. This intuition is confirmed in the next proposition, which invokes the following two conditions:

Condition R1: For any $p \geq p'$ and $s \geq s'$,

$$\frac{\psi_1(p + s)}{p} - \frac{\psi_1(p' + s)}{p'} \leq \frac{\psi_1(p + s')}{p} - \frac{\psi_1(p' + s')}{p'}.$$

This condition means that firm 1's equilibrium prize bid is nondecreasing in its investment in an auction without a binding maximum prize. Technically, it requires that ψ_1 not be too convex. It is satisfied by cost functions of the form $\psi_1(x) = \alpha x^\gamma$, with $0 < \alpha$ and $0 < \gamma \leq 2$, for example.

Condition R2: Either

(i) $$\frac{\psi_1(x)}{\psi_1'(x)} \leq \frac{\psi_2(x')}{\psi_2'(x')} \quad \forall x < x',$$

or

(ii) $$\frac{\psi_1(x)}{\psi_1'(x)} > \frac{\psi_2(x')}{\psi_2'(x')} \quad \forall x < x'$$

and $\psi_1(\cdot)$ is convex.

This condition requires that the growth rates of the firms' costs be uniformly ranked (the rate of increase of firm 1's cost is uniformly higher or lower than firm 2's). Condition R2(i) is satisfied by cost functions of the form $\psi_1(x) = \alpha x^\gamma$ and $\psi_2(x) = \beta x^\gamma$, with $0 < \alpha < \beta$ and $\gamma > 0$, for example. Condition R2(ii) simply requires that firm 1's costs be convex and have a lower growth rate along with the lower level.

PROPOSITION 6: *Given Conditions R1 and R2, it is optimal for the buyer to invite firms 1 and 2 to participate in a first-price auction, and to set a maximum allowable prize of $\bar{p}^*$ for firm 1.*

The optimal contest for a buyer facing asymmetric firms is again an auction. She will invite

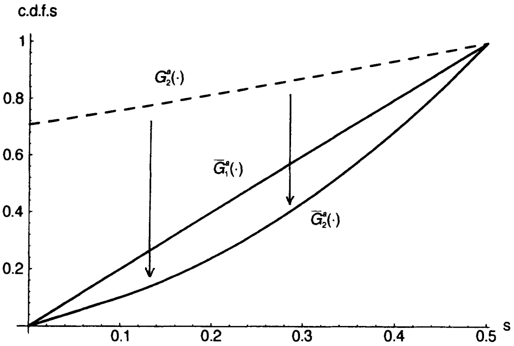

FIGURE 2. THE EFFECT OF HANDICAPPING

the two most efficient firms to participate, and she will handicap the more efficient one so that it receives no rents. The handicapping causes the less efficient firm to compete more aggressively, offering even greater surplus than the more efficient one does, on average. This is illustrated by the next example.

Example 6 (*Optimal auction with handicapping*): Revisit Example 3 with $\psi_1(x) = 1/4\, x^2$ and $\psi_2(x) = 1/2\, x^2$. The optimal auction imposes a maximum allowable prize of $\bar{p}^* = 3/2 - \sqrt{2}$ on firm 1. The firms then offer surplus in [0, 1/2] according to the c.d.f.s

$$\bar{G}_1^a(s) = 2s \text{ and}$$

$$\bar{G}_2^a(s) = \begin{cases} s & \text{if } s \in [0, \bar{p}^*], \\ \dfrac{(s + \bar{p}^*)^2}{4\bar{p}^*} & \text{if } s \in \left[\bar{p}^*, \dfrac{1}{2}\right], \end{cases}$$

which are graphed in Figure 2. Absent handicapping, firm 1 adopts $G_1^a(\cdot) = \bar{G}_1^a(\cdot)$ while firm 2 uses $G_2^a(\cdot)$ (the dotted curve in Figure 2). The handicapping of firm 1 shifts firm 2's c.d.f. down to $\bar{G}_2^a(\cdot)$, which is below $\bar{G}_1^a(\cdot)$. In other words, the handicapping makes firm 2 compete more aggressively (i.e., offer higher surplus) than firm 1.

IV. Conclusion

This paper has studied an important set of policy variables associated with the design of contests to procure an innovation. In particular, we have compared various methods for selecting the winning contestant and determining the prize, including first-price auctions and fixed-prize tournaments. Our results show that letting the two most efficient innovators participate and bid for their prizes is optimal. Handicapping the more efficient one through imposition of a maximum allowable prize is optimal when contestants are asymmetric.

Our analysis has useful implications beyond research contest design. First, our model has clear connections to the optimal auction design problem (for comprehensive surveys, see Milgrom, 1985; McAfee and John McMillan, 1987; Paul Klemperer, 2000). The auction literature studies the allocation of resources based on bidding, much as in our model. The novel element here is that agents' types are determined endogenously through their investment decisions. This feature made the revenue equivalence theorem inapplicable, and it yielded a nontrivial comparison among mechanisms with (*ex post*) efficient allocations. While the desirability of using auctions and of handicapping parallels findings in the auction literature, there are also significant differences. These include the optimality of choosing only two firms and the lack of a binding reserve in the symmetric case. Bidders in many real-world auctions have opportunities to make investments that will influence the value of auctions, making this line of inquiry particularly important.[33]

Our findings also shed light on how to deal with the holdup problem that arises in contracting relationships. The key source of the holdup problem is the lack of verifiability of contractors' efforts, which makes them susceptible to expropriation by their contract partners. That bilateral contracts are often inadequate for protecting specific investments

[33] Several authors have studied auctions with prebidding investment (see Kenneth French and Robert McCormick, 1984; Kevin Lang and Robert Rosenthal, 1991; Ian King et al., 1992; Guofu Tan, 1992; Levin and Smith, 1994; Parimal Bag, 1997; Nicola Persico, 2000; Dirk Bergemann and Juuso Välimäki, 2000). Except for the last paper, which searches for a *welfare*-maximizing mechanism when investments take the form of information acquisition, this literature simply assumes the use of auctions. Moreover, none of these papers considers cooperative investments, a feature of much R&D procurement.

has led many authors to explore organizational remedies such as vertical integration (Benjamin Klein et al., 1978; Oliver Williamson, 1979), shifting property rights (Sanford Grossman and Oliver Hart, 1986; Hart and John Moore, 1990), and allocating financial and nonfinancial decision rights (Philippe Aghion and Patrick Bolton, 1992; Aghion and Jean Tirole, 1997). Our results imply that introducing competition among potential contract partners can provide incentives for effort that may be difficult to motivate through bilateral contracts. While this "incentivizing" effect of competition has been noted recently,[34] a novel implication of the current study is that auctions may be particularly good at harnessing this effect.

The current paper has focused on a deterministic environment in which each firm has complete control over the quality of its innovation. In practice, the outcome of innovative activity is likely to be uncertain, but this need not detract from the desirability of contests. If the uncertainty is not too great, the suppliers will still randomize over quality, and auctions are likely to remain desirable. Indeed, Fullerton et al. (2002) have shown that the desirability of auctions is robust to stochastic innovation technologies. A general analysis involving uncertainty seems nontrivial, however, since our duality arguments do not readily generalize to the case of uncertain outcomes.

[34] See W. Bentley MacLeod and James M. Malcomson (1993), Harold Cole et al. (1998), Che and Hausch (1999), and Leonardo Felli and Kevin Roberts (2001). These papers focus on perfect-information settings (i.e., all parties can observe the levels of investment), and they do not explore mechanism design.

APPENDIX

PROOF OF LEMMA 6:

When a firm offers $s > 0$, it must choose a positive quality, which can only be profitable if the firm asks for a prize satisfying $p > 0$. Also, by Lemma 2, firm i wins with probability $\Pi_{j\in\mathcal{N}\setminus\{i\}} G_j(s)$ since ties occur with probability zero when $s > 0$. These facts imply that, for any $s \in S_i \cap \Re_{++}$,

$$u_i = \sup_{p\in\mathcal{P}_i}\left\{p \prod_{j\in\mathcal{N}\setminus\{i\}} G_j(s) - \psi_i(s+p)\right\}$$

$$\Leftrightarrow 0 = \sup_{p\in\mathcal{P}_i}\left\{\prod_{j\in\mathcal{N}\setminus\{i\}} G_j(s) - \frac{u_i + \psi_i(s+p)}{p}\right\}$$

$$\Leftrightarrow \prod_{j\in\mathcal{N}\setminus\{i\}} G_j(s) = \inf_{p\in\mathcal{P}_i}\left\{\frac{u_i + \psi_i(s+p)}{p}\right\},$$

which is equivalent to (2). Clearly, the infimum is attainable if and only if the supremum of (1) is attainable, in which case the set of minimizers coincides with the set of maximizers of (1). To prove the last statement, suppose firm i chooses (x, p) that does not minimize (2). By duality, (1) will fail to hold, contradicting the assertion that firm i earns u_i and $s \in S_i$.

PROOF OF PROPOSITION 1:

(*Characterization*) This part of the proof has several steps.

Step 1: In equilibrium, at most one firm receives a strictly positive expected payoff.

PROOF:

Suppose, to the contrary, that both firms receive a strictly positive expected payoff. The firms must have a common infimum surplus; if not, the firm with the lower infimum would get a nonpositive expected payoff when offering surplus between the two infima since such an offering cannot win. Moreover, the firms must place mass on the common infimum, and the corresponding prize bid must be strictly positive. (If fewer than two put mass there, at least one firm would get an expected payoff arbitrarily close to zero when offering surplus arbitrarily close to the infimum.) Each of these firms would then have an incentive to deviate by lowering its prize bid infinitesimally while holding its quality fixed. Such a deviation is profitable since the probability of winning would then jump up. Hence, at most one firm can receive a strictly positive expected payoff.

Step 2: In equilibrium, $\bar{s}_k = s_j^*(\bar{p}_j)$ for $k = i, j$.

PROOF:

By Lemma 4, it suffices to show that the overall supremum satisfies $\bar{s} = s_j^*(\bar{p}_j)$. Suppose, first, that $\bar{s} > s_j^*(\bar{p}_j)$. Since $s_j^*(\bar{p}_j)$ is the most that firm j can offer in any equilibrium, only firm i must be offering $s \in (s_j^*(\bar{p}_j), \bar{s}]$. This last point contradicts Lemma 4, so $\bar{s} \leq s_j^*(\bar{p}_j)$. Now suppose $\bar{s} < s_j^*(\bar{p}_j)$. For each $k = i, j$, there exists a feasible $(\tilde{x}_k, \tilde{p}_k)$ such that $\tilde{x}_k - \tilde{p}_k \in (\bar{s}, s_j^*(\bar{p}_j)]$ and $\tilde{p}_k - \psi_k(\tilde{x}_k) \geq 0$. [This holds for $k = j$ by definition of $s_j^*(\bar{p}_j)$ and for $k = i$ since $s_i^*(\bar{p}_i) \geq s_j^*(\bar{p}_j)$ and $\psi_i(x)$ is continuous for $x > 0$.] By offering a quality slightly below $\tilde{x}_k$ and asking for $\tilde{p}_k$, firm k can still offer $s \in (\bar{s}, s_j^*(\bar{p}_j)]$. With that offer, firm k wins with probability 1 and earns a strictly positive expected payoff since $\psi_k(x) < \psi_k(\tilde{x}_k) \leq \tilde{p}_k$. This argument holds for $k = i, j$, so we have a contradiction to Step 1.

Step 3: In equilibrium, firm j earns an expected payoff of zero and firm i earns $u_i^*(\bar{p}_i)$.

PROOF:

Firm i could offer a net surplus of $s_j^*(\bar{p}_j)$ and earn $u_i^*(\bar{p}_i)$, so $u_i \geq u_i^*(\bar{p}_i)$. For firm i to earn more than $u_i^*(\bar{p}_i)$ requires $\bar{s} < s_j^*(\bar{p}_j)$. But this would contradict Step 2, so firm i receives $u_i^*(\bar{p}_i)$. If $u_i^*(\bar{p}_i) = 0$, which means $s_i^*(\bar{p}_i) = s_j^*(\bar{p}_j)$, the symmetric argument proves that firm j receives an expected payoff of zero. If $u_i^*(\bar{p}_i) > 0$, the same conclusion is reached via Step 1.

Step 4: In equilibrium, $\underline{s}_i = 0$.

PROOF:

Suppose not. Then, by Lemma 5, $\underline{s}_i < 0$. If firm j were to choose zero quality and a prize $p_j \in (0, -\underline{s}_i)$, it would win with positive probability under (R′), since $-p_j > \underline{s}_i$. It would then earn a strictly positive expected payoff, which contradicts Step 3.

Step 5: In equilibrium, $S_i = [0, s_j^*(\bar{p}_j)]$, $S_j \cap \Re_+ = [0, s_j^*(\bar{p}_j)]$, and firm k offers surplus $s \in (0, s_j^*(\bar{p}_j)]$ according to $G_k(s)$ described in the statement of the proposition, for $k = i, j$.

PROOF:

By Lemmas 4 and 5, and by Step 2 above, $S_k \cap \Re_+ = [0, s_j^*(\bar{p}_j)]$. Step 4 then implies $S_i = [0, s_j^*(\bar{p}_j)]$. Furthermore, by Step 3, $u_i = u_i^*(\bar{p}_i)$ and $u_j = 0$. Therefore, by Lemma 6, the c.d.f. of firm k's surplus offer must equal $G_k(s)$, as described in the statement of the proposition, for almost every $s \in (0, s_j^*(\bar{p}_j)]$. Since the c.d.f. is nondecreasing and the minimized value, $G_k(s)$, is continuous (by the Theorem of the Maximum), the former equals the latter for every $s \in (0, s_j^*(\bar{p}_j)]$.

(*Existence*) We now establish existence of an equilibrium. Consider the following strategy profile:

$$G_i(s) := \begin{cases} 1 & \text{for } s \geq s_j^*(\bar{p}_j), \\ \min_{p \in [0, \bar{p}_j]} \dfrac{\psi_j(p+s)}{p} & \text{for } s \in (0, s_j^*(\bar{p}_j)], \\ \inf_{p \in (0, \bar{p}_j]} \dfrac{\psi_j(p)}{p} & \text{for } s = 0, \\ 0 & \text{for } s < 0. \end{cases}$$

and

$$G_j(s) := \begin{cases} 1 & \text{for } s \geq s_j^*(\bar{p}_j), \\ \min_{p \in [0, \bar{p}_i]} \dfrac{u_i^*(\bar{p}_i) + \psi_i(p+s)}{p} & \text{for } s \in (0, s_j^*(\bar{p}_j)], \\ \inf_{p \in (0, \bar{p}_i]} \dfrac{u_i^*(\bar{p}_i) + \psi_i(p)}{p} & \text{for } s = 0, \\ 0 & \text{for } s < 0. \end{cases}$$

These c.d.f.s coincide with $G_i(\cdot)$ and $G_j(\cdot)$, as stated in the proposition, for $s \in (0, s_j^*(\bar{p}_j)]$. The corresponding quality-prize pair is given by the minimizer $(x_k^a(s), p_k^a(s))$, which is well defined for each $s \in (0, s_j^*(\bar{p}_j)]$. The pair is also well defined for $s = 0$ if $k = i$ and $u_i^*(\bar{p}_i) > 0$. For the remaining case with $s = 0$, the corresponding strategy is $x_k^a(s) = p_k^a(s) = 0$. Under (R′), firms i and j will earn $u_i^*(\bar{p}_i)$ and 0, respectively, by playing these strategies.

We now show that any unilateral deviation is unprofitable. Suppose that firm k deviates to (x, p), with $p \in [0, \bar{p}_k]$. If $x < p$, (x, p) will win with zero probability, so such a deviation is unprofitable. Conversely, if $x - p > s_j^*(\bar{p}_j)$, the pair is strictly dominated by $(x - \varepsilon, p)$, for small $\varepsilon > 0$. Likewise, a deviation is unprofitable if $p = 0$. We can therefore restrict attention to deviations (x, p) with $x - p \in [0, s_j^*(\bar{p}_j)]$ and $p > 0$. Letting $u_k = 0$ if $k = j$ and $u_k = u_i^*(\bar{p}_i)$ if $k = i$, such a deviation implies

$$\Pr\{\text{firm } k \text{ wins with } (x, p)\} \leq G_{-k}(x - p) = \inf_{p' \in (0, \bar{p}_k]} \frac{u_k + \psi_k(x - p + p')}{p'} \leq \frac{u_k + \psi_k(x)}{p},$$

where the first inequality holds since the other firm may put mass at $s = 0$,[35] the equality follows from the definition of G_{-k},[36] and the last inequality follows since the infimum need not be attained at $p' = p$. The above string of inequalities implies

$$p \Pr\{\text{firm } k \text{ wins with } (x, p)\} - \psi_k(x) \leq u_k,$$

so it does not pay firm $k = i, j$ to deviate.

[35] The first inequality holds with equality for firm i, and it holds with equality for firm j for $s \in (0, s_j^*(\bar{p}_j)]$.

[36] The definition is precise if $x - p = 0$. For $x - p > 0$, the infimum is attained at some $p' \in \mathcal{P}_i \setminus \{0\}$, so $\inf_{p' \in (0, \bar{p}_k]} \dfrac{u_k + \psi_k(x - p + p')}{p'} = \min_{p' \in [0, \bar{p}_k]} \dfrac{u_k + \psi_k(x - p + p')}{p'}$.

PROOF OF PROPOSITION 2:

We first prove (*i*). Let S_i^a denote the support of the surplus offer from firm $i \in \mathcal{N}$ in that equilibrium. We first show that $x_i^a(s) < x_i^a(s')$ for surpluses s and $s' > s$ in $S_i^a \cap \mathfrak{R}_+$. Suppose, to the contrary, that $x_i^a(s) \geq x_i^a(s')$. Since $s' > 0$, we must have $x_i^a(s') > 0$, implying $x_i^a(s) > 0$ as well. Hence, $x_i^a(s)$ solves (2) for $\mathcal{P}_i = \mathfrak{R}_+$. Then, footnote 25 implies $x_i^a(s) \leq x_i^a(s')$. Therefore, $x_i^a(s) = x_i^a(s') = x$, for some $x > 0$. In light of Condition D (differentiability of ψ_i), x must satisfy the first-order conditions:

$$\frac{\psi_i'(x)[x - s] - u_i(\infty) - \psi_i(x)}{(x - s)^2} = 0 \text{ and } \frac{\psi_i'(x)[x - s'] - u_i(\infty) - \psi_i(x)}{(x - s')^2} = 0.$$

Since $x > \max\{s, s'\}$, the two equalities imply

$$\psi_i'(x)[x - s] = u_i(\infty) + \psi_i(x) = \psi_i'(x)[x - s'],$$

which cannot hold for any $x > 0$. We conclude that $x_i^a(s)$ is strictly increasing for all $s \in S_i^a \cap \mathfrak{R}_+$.

We now prove that no firm selects any particular quality with positive probability. Suppose, to the contrary, that firm i puts positive mass on a particular quality, $x \in X_i^a \backslash \{0\}$. Since $x_i^a(\cdot)$ is strictly increasing, there exists a unique $s \geq 0$ such that $x = x_i^a(s)$.[37] By Lemma 2, a firm cannot put mass on $x = x_i^a(s)$ for some $s > 0$, so it must be that $x = x_i^a(0)$. (That is, if the firm puts mass on x, it must be offering zero surplus.) This can only be profitable if firm $j \neq i$ offers $s \leq 0$ with positive probability. It follows that both firms must offer $s \leq 0$ with positive probability, but this cannot happen in equilibrium either. Since $\psi_i'(0) = 0$ (by Condition D), each firm could choose $x = \varepsilon > 0$ and offer $p = \varepsilon/2$, which will win with positive probability and yield a strictly positive expected payoff, for sufficiently small ε. Hence, both must earn a strictly positive expected payoff, but this contradicts Corollary 1. We conclude that $X_i^a \backslash \{0\}$ contains no mass points.

The equilibrium bidding strategies are derived from the first-order condition,

$$\frac{\psi_i'(x)[x - s] - u_i(\infty) - \psi_i(x)}{(x - s)^2} = 0,$$

and the strict monotonicity of $x_i^a(\cdot)$:

$$b_i(x) = x - x_i^{a^{-1}}(x) = \frac{u_i(\infty) + \psi_i(x)}{\psi_i'(x)}.$$

We next prove (*ii*). If $\psi_i(\cdot)$ is strictly convex, the minimizer of (5), $x_i^a(s)$, is unique for $s \in (0, s_2^*]$, so $x_i^a(\cdot)$ is continuous, by Berge's Theorem of the Maximum. Since $S_i^a \cap \mathfrak{R}_{++}$ is an interval (by Lemma 3), $X_i^a = \{z \mid z = x_i^a(s) \text{ for some } s \in (0, s_2^*]\}$ is then an interval. It now suffices to show that $x_i^a(s)$ is continuous at $s = 0$ whenever $x_i^a(0)$ is in the support of i's quality choice. If $x_i^a(0) > 0$, then $x_i^a(0)$ is a minimizer of (5), so continuity follows from the above argument. Hence, let $x_i^a(0) = 0$ and suppose, to the contrary, that $x_+ := \lim_{s \downarrow 0} x_i^a(s) > 0$. For $x_i^a(0)$ to be part of firm i's support, the firm must put mass on that quality. Moreover, we must have $u_i^a = 0$. This last assertion follows from Proposition 1 if $i = 2$. If $i = 1$, it follows since the firm never offers negative surplus (again by Proposition 1), so it must choose $x_i^a(0) + 0 = 0$ with positive probability. Since $x_i^a(s)$ is a minimizer of (5) for all $s > 0$, we must have $\psi_i'(x_i^a(s))[x_i^a(s) + s] \leq \psi_i(x_i^a(s))$, where

[37] A firm will choose $x > 0$ in equilibrium only if it will win with positive probability. By Corollary 1, no firm wins with positive probability when offering $s < 0$.

ψ_i' is a right derivative. As $s \to 0$, we have $\psi_i'(x_+)x_+ \leq \psi_i(x_+)$. But this last fact contradicts strict convexity of $\psi_i(\cdot)$.

PROOF OF PROPOSITION 3:

To prove statement (*i*), let

$$X^a(s) := \left\{ x : x \in \arg\min_{x \geq 0} \frac{\psi(x)}{x - s} \right\}.$$

As is argued in footnote 25, every selection of $X^a(s)$ is nondecreasing in s. Hence, $X^a(s)$ is a singleton for almost every s. Moreover, by Lemma 2, there are no mass points in surplus offerings for $s > 0$. Given the monotonicity of $X^a(s)$, the firm that chooses the higher $x > 0$ wins the contest with probability 1 when the firms are symmetric.

We now prove statement (*ii*). Fix an equilibrium, and let S_1^a be the support of firm 1's net surplus offering. Invoking the argument above, we can restrict attention to $s \in S_1^a$ such that $\frac{s_1^* - s_2^* + \psi_1(x)}{x - s}$ and $\frac{\psi_2(x)}{x - s}$ have unique minimizers, $x_1^a(s)$ and $x_2^a(s)$, respectively.

For $s \in S_1^a \cap \Re_{++}$, let

$$\rho(x, z) := -z \frac{s_1^* - s_2^* + \psi_1(x)}{x - s} - (1 - z) \frac{\psi_2(x)}{x - s}.$$

Clearly, $x_{2-z}^a \in \arg\max_{x \geq 0} \rho(x, z)$ and $x_{2-z}^a > s$, for $z = 0, 1$. Now observe that $\rho(x, z)$ satisfies the strict single-crossing property in (x, z) for any $x > s$. By Theorem 4′ of Milgrom and Shannon (1994), we then have $x_1^a(s) \geq x_2^a(s)$ for any $s \in S_1^a \cap \Re_{++}$. This inequality implies that firm 1 asks for a weakly higher prize, given the same x, which proves the first statement of (*ii*). The second statement of (*ii*) is proven by showing that the first-order conditions cannot both hold at a given $x > 0$, much as in the proof of Proposition 2.

PROOF OF PROPOSITION 4:

By Corollary 1, we have

$$\hat{G}_1^a(s) = \min_{x \geq s} \frac{\psi_1(x)}{x - s}, \; G_1^a(s) = \hat{G}_2^a(s) = \min_{x \geq s} \frac{\psi_2(x)}{x - s}, \text{ and } G_2^a(s) = \min_{x \geq s} \frac{s_1^* - s_2^* + \psi_1(x)}{x - s},$$

for $s \in (0, s_2^*)$. Hence, it suffices to show that

$$\min_{x \geq s} \frac{\psi_1(x)}{x - s} < \min_{x \geq s} \frac{\psi_2(x)}{x - s} \leq \min_{x \geq s} \frac{s_1^* - s_2^* + \psi_1(x)}{x - s}.$$

The first inequality follows since $\psi_1(\cdot) < \psi_2(\cdot)$. To see the second inequality, suppose $G_2^a(s) < G_1^a(s)$. Since $G_2^a(s_2^*) = G_1^a(s_2^*) = 1$, we can use the Mean Value Theorem [applied to $G_2^a(\cdot) - G_1^a(\cdot)$] to show that there exists $s' \in (s, s_2^*)$ such that $G_2^a(s') < G_1^a(s')$ and $G_2^{a\prime}(s') > G_1^{a\prime}(s')$. Meanwhile, the Envelope Theorem implies

$$G_2^{a\prime}(s') - G_1^{a\prime}(s') = \left[\frac{G_2^a(s')}{x_1^a(s') - s'} - \frac{G_1^a(s')}{x_2^a(s') - s'} \right] < 0,$$

where the inequality holds since $G_2^a(s') < G_1^a(s')$ and $x_1^a(s') \geq x_2^a(s')$, by Proposition 3. [It shows that $x_1 = x_2$ implies $p_1 \geq p_2$. Since $x_k^a(s)$ is nondecreasing, we have $x_1^a(s) \geq x_2^a(s)$.] We thus have a contradiction. If Condition D also holds, then $x_1^a(s') > x_2^a(s')$ [recall the proof of Proposition 3(*ii*)], so the same argument proves that $G_2^a(s) > G_1^a(s)$.

PROOF OF PROPOSITION 6:

The proof requires the following two lemmas.

LEMMA 7: *Let $\mathcal{P}_1$ and u_1 be firm 1's menu of prizes and its equilibrium expected payoff, respectively, in an arbitrary contest. Given Condition R1, for any $s \in [0, s_2^*]$ we have*

$$\bar{G}_2^a(s) \leq \inf_{p \in \mathcal{P}_1} \frac{u_1 + \psi_1(p + s)}{p}.$$

PROOF:

Recall first that $u_1^*(\bar{p}) := \max_{p \in [0,\bar{p}]} p - \psi_1(p + s_2^*)$. Now let $\hat{p}$ be the smallest value of $\bar{p} \in [\bar{p}^*, x_1^* - s_2^*]$ that satisfies

$$u_1^*(\bar{p}) = \max\{\sup_{p \in \mathcal{P}_1} p - \psi_1(p + s_2^*), 0\}. \tag{A1}$$

The proof involves several steps.

Step 1: $u_1 \geq u_1^*(\hat{p})$.

PROOF:

Let $\bar{s}$ be the supremum surplus in the equilibrium. By Lemma 5, $\bar{s} \leq s_2^*$. Since firm 1 can win with probability 1 by offering $\bar{s}$ (it can choose quality $p + \bar{s}$ and bid p, for some $p \in \mathcal{P}_1$), its equilibrium expected payoff satisfies

$$u_1 \geq \sup_{p \in \mathcal{P}_1} p - \psi_1(p + \bar{s}) \geq \sup_{p \in \mathcal{P}_1} p - \psi_1(p + s_2^*).$$

The result follows from (A1) and the requirement that $u_1 \geq 0$ in equilibrium.

Step 2: $\inf_{p \in \mathcal{P}_1} \dfrac{u_1^*(\hat{p}) + \psi_1(p + s)}{p} \geq \min_{p \in [0,\hat{p}]} \dfrac{u_1^*(\hat{p}) + \psi_1(p + s)}{p}$.

PROOF:

Given Condition R1, $\dfrac{u + \psi_1(p + s)}{p}$ is submodular in (p, s) for any u. Hence, for any $p' \in \mathcal{P}_1 \backslash [0, \hat{p}]$ and $s \leq s_2^*$, we have

$$\frac{u_1^*(\hat{p}) + \psi_1(\hat{p} + s)}{\hat{p}} - \frac{u_1^*(\hat{p}) + \psi_1(p' + s)}{p'} \leq \frac{u_1^*(\hat{p}) + \psi_1(\hat{p} + s_2^*)}{\hat{p}} - \frac{u_1^*(\hat{p}) + \psi_1(p' + s_2^*)}{p'} \leq 0,$$

where the last inequality follows since $u_1^*(\hat{p}) \geq p' - \psi_1(p' + s_2^*)$ for any $p' \in \mathcal{P}_1 \backslash [0, \hat{p}]$ (otherwise, we have a contradiction to the definition of $\hat{p}$). The above inequality then implies the result.

Step 3: For any $\bar{p} \geq \bar{p}^*$, we have

$$\min_{p \in [0,\bar{p}]} \frac{u_1^*(\bar{p}) + \psi_1(p+s)}{p} \geq \min_{p \in [0,\bar{p}^*]} \frac{\psi_1(p+s)}{p}.$$

PROOF:
Let

$$h(\bar{p}, s) := \min_{p \in [0,\bar{p}]} \frac{u_1^*(\bar{p}) + \psi_1(p+s)}{p}.$$

Since $u_1^*(\cdot)$ is nondecreasing, the negative of the minimand (the objective function) satisfies increasing differences in $(p; \bar{p})$, so Theorem 4 of Milgrom and Shannon (1994) implies that we can select a minimizer, $z(\bar{p}, s)$, that is nondecreasing in $\bar{p}$. For the proof, it suffices to show $h(\bar{p}', s) \geq h(\bar{p}^*, s)$ for $\bar{p}' > \bar{p}^*$, given $s \in (0, s_2^*]$. To that end, note first that

$$h(\bar{p}', s_2^*) - h(\bar{p}', s) = \int_s^{s_2^*} \frac{\psi_1'(z(\bar{p}', t) + t)}{z(\bar{p}', t)}\, dt \leq \int_s^{s_2^*} \frac{\psi_1'(z(\bar{p}^*, t) + t)}{z(\bar{p}^*, t)}\, dt$$

$$= h(\bar{p}^*, s_2^*) - h(\bar{p}^*, s),$$

where the equalities hold by the Envelope Theorem since $\psi_1(x)$ is absolutely continuous for $x > 0$ [see Theorems 1 and 2 of Milgrom and Ilya Segal (2002)] and the inequality follows from Condition R1 since $z(\bar{p}', t) \geq z(\bar{p}^*, t)$. The result then follows since $h(\bar{p}', s_2^*) = h(\bar{p}^*, s_2^*) = 1$ [by the definition of $u_1^*(\cdot)$], given $\bar{p}' > \bar{p}^*$.

Step 4: For any $s \in [0, s_2^*]$,

$$\bar{G}_2^a(s) \leq \inf_{p \in \mathcal{P}_1} \frac{u_1 + \psi_1(p+s)}{p}.$$

PROOF:
The result holds since

$$\inf_{p \in P_1} \frac{u_1 + \psi_1(p+s)}{p} \geq \inf_{p \in P_1} \frac{u_1^*(\hat{p}) + \psi_1(p+s)}{p} \geq \min_{p \in [0,\hat{p}]} \frac{u_1^*(\hat{p}) + \psi_1(p+s)}{p}$$

$$\geq \min_{p \in [0,\bar{p}^*]} \frac{\psi_1(p+s)}{p} = \bar{G}_2^a(s),$$

where the inequalities follow from Steps 1, 2, and 3, respectively, and the equality follows from the definition.

LEMMA 8: *Given Condition R2, we have* $\bar{G}_2^a(s) \leq \bar{G}_1^a(s)$ *for any* $s \in [0, s_2^*]$.

PROOF:

The proof again comprises several steps. Let $p_2^a(s) \in \arg\min_{p \geq 0} \frac{\psi_2(p+s)}{p}$ and let $p^*(s) \in \arg\min_{p \in [0, \bar{p}^*]} \frac{\psi_1(p+s)}{p}$.

Step 1: If $\bar{G}_2^a(s) > \bar{G}_1^a(s)$, then $p^*(s) < p_2^a(s)$.

PROOF:

Suppose $p_2^a(s) \leq p^*(s)$ for some $s \in (0, s_2^*]$. For such a surplus offering, we have

$$\bar{G}_2^a(s) = \min_{p \in [0, \bar{p}^*]} \frac{\psi_1(p+s)}{p} \leq \frac{\psi_1(p_2^a(s)+s)}{p_2^a(s)} \leq \frac{\psi_2(p_2^a(s)+s)}{p_2^a(s)} = \bar{G}_1^a(s),$$

where the first inequality holds since $p_2^a(s) \leq p^*(s) \leq \bar{p}^*$. Since $p_2^a(s) \leq p^*(s)$ implies $\bar{G}_2^a(s) \leq \bar{G}_1^a(s)$, the contrapositive also holds.

Step 2: Given Condition R2(i), $\bar{G}_2^a(s) \leq \bar{G}_1^a(s)$ for all $s \in [0, s_2^*]$.

PROOF:

Suppose, to the contrary, that $\bar{G}_2^a(s) > \bar{G}_1^a(s)$ for some $s \in [0, s_2^*)$. Both c.d.f.s are continuous (by the Theorem of the Maximum) and they satisfy $\bar{G}_2^a(s_2^*) = \bar{G}_1^a(s_2^*) = 1$.[38] The Mean Value Theorem implies that there exists $s' \in [s, s_2^*]$ such that $\bar{G}_2^a(s') > \bar{G}_1^a(s')$ and $\bar{G}_2^{a\prime}(s') \leq \bar{G}_1^{a\prime}(s')$. Meanwhile, Step 1 implies that $p^*(s') < p_2^a(s')$. Hence,

$$\bar{G}_1^{a\prime}(s') = \left[\frac{\psi_2'(p_2^a(s')+s')}{\psi_2(p_2^a(s')+s')}\right]\bar{G}_1^a(s') < \left[\frac{\psi_1'(p^*(s')+s')}{\psi_1(p^*(s')+s')}\right]\bar{G}_2^a(s') = \bar{G}_2^{a\prime}(s'),$$

where the equalities follow from the Envelope Theorem and the inequality follows from $\bar{G}_2^a(s') > \bar{G}_1^a(s')$ and $p^*(s') < p_2^a(s')$, and from Condition R2(i). Since the inequality yields a contradiction, we conclude that $\bar{G}_2^a(s) \leq \bar{G}_1^a(s)$ for all $s \in [0, s_2^*]$.

Step 3: Given Condition R2(ii), $\bar{G}_2^a(s) \leq \bar{G}_1^a(s)$ for all $s \in [0, s_2^*]$.

Since $\psi_1(\cdot)$ is convex and $\psi_1(0) = 0$, it follows that $\frac{\psi_1(p)}{p}$ is nondecreasing for all $p > 0$. This means the maximum bid of $\bar{p}^* > 0$ is not binding when $s = 0$. Hence, $\bar{G}_2^a(0) = \min_{p \in [0, \bar{p}^*]} \frac{\psi_1(p)}{p} \leq \min_{p \in \Re_+} \frac{\psi_2(p)}{p} = \bar{G}_1^a(0)$. Now suppose, contrary to the claim, that $\bar{G}_2^a(s) > \bar{G}_1^a(s)$ for some $s \in (0, s_2^*)$. Then, there must exist $s' \in [0, s)$ such that $\bar{G}_2^a(s') \leq \bar{G}_1^a(s')$ and $\bar{G}_2^{a\prime}(s') \geq \bar{G}_1^{a\prime}(s')$. An argument analogous to that in Step 2 then produces a contradiction, given Condition R2(ii).

We now prove the proposition. Consider an arbitrary contest with $|\mathcal{N}| \geq 2$, and fix an equilibrium. Suppose firm $i \in \mathcal{N}$ faces a menu of prizes given by $\mathcal{P}_i$, offers surplus according to some c.d.f. G_i, and receives an expected payoff $u_i \geq 0$ in that equilibrium. Consider any $s \in (0, s_2^*]$. By Lemma 4, at least two firms are active at that surplus level. Suppose firms $i, j \in \mathcal{N}$ are active, $i < j$. It follows from Lemma 6 that

[38] Note that $\bar{G}_2^a(s_2^*) = 1$, by the definition of $\bar{p}^*$, and $\bar{G}_1^a(s_2^*) = 1$, by the definition of s_2^*.

$$\text{(A2)} \quad \prod_{k \in \mathcal{N}\setminus\{j\}} G_k(s) = \inf_{p \in \mathcal{P}_j} \frac{u_j + \psi_j(p+s)}{p} \geq \inf_{p \in \mathcal{P}_j} \frac{\psi_j(p+s)}{p} \geq \min_{p \geq 0} \frac{\psi_2(p+s)}{p} = \bar{G}_1^a(s),$$

where the last inequality holds since $j \geq 2$. Similarly,

$$\text{(A3)} \quad \prod_{k \in \mathcal{N}\setminus\{i\}} G_k(s) = \inf_{p \in P_i} \frac{u_i + \psi_i(p+s)}{p} \geq \min\left\{\inf_{p \in P_1} \frac{u_1 + \psi_1(p+s)}{p}, \min_{p \geq 0} \frac{\psi_2(p+s)}{p}\right\}$$

$$\geq \min_{p \in [0,\bar{p}^*]} \frac{\psi_1(p+s)}{p} = \bar{G}_2^a(s),$$

where the first inequality is immediate for the cases of $i = 1$ and $i \geq 2$ both, and the second inequality follows from Lemmas 7 and 8.

By (A2) and (A3), we then have

$$\left(\prod_{k \in \mathcal{N}} G_k(s)\right)\left(\prod_{k \in \mathcal{N}\setminus\{i,j\}} G_k(s)\right) \geq \bar{G}_1^a(s)\bar{G}_2^a(s).$$

Since $G_k(s) \in (0, 1]$, it follows that

$$\prod_{k \in \mathcal{N}} G_k(s) \geq \bar{G}_1^a(s)\bar{G}_2^a(s).$$

Since s was arbitrary, the net surplus accruing to the buyer under the auction stochastically dominates the surplus from the arbitrary contest. This proves the proposition.

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