Caps on Political Lobbying

By YEON-KOO CHE AND IAN L. GALE*

The cost of political campaigns in the United States has risen substantially in recent years. For example, real spending on congressional election campaigns doubled between 1976 and 1992 (Steven D. Levitt, 1995). There are many reasons why increased campaign spending might be socially harmful. First, increased spending means increased fund-raising, which may keep politicians from their legislative duties.[1] Second, a lobbyist who makes a large campaign contribution may have undue influence on electoral outcomes, on the shaping of legislation, or on the outcome of regulatory proceedings.[2] That is, the socially preferred candidate or legislation may not prevail. Likewise, a lobbyist involved in a regulatory matter or a competition for a government contract may benefit unduly from a legislator's intervention.[3] Third, a perception that campaign contributions purchase influence may lead to increased tolerance of corruption in the private sector.

A desire to control campaign spending has spawned many initiatives to limit both campaign contributions and spending, beginning with the passage of the Federal Election Campaign Act (FECA). Political action committees can contribute at most $5,000 per election to a candidate, while individuals can contribute at most $1,000. (Restrictions have also been put on in-kind contributions, making it more difficult to circumvent these limits.)[4] While direct restrictions on campaign spending have proven difficult to implement, recent initiatives aim to impose voluntary spending limits and stricter limits on contributions.[5]

Despite the existing legislation and the proposals to limit contributions, little is known about the impact of contribution limits on aggregate expenditures. While it is intuitively appealing that aggregate expenditures would drop, we challenge that intuition here. We study a lobbying game and show that a cap on individual lobbyists' expenditures may have the perverse effect of increasing aggregate expenditures and lowering total surplus. This result suggests that a cap on campaign contributions may increase aggregate contributions.[6]

* Che: Department of Economics, University of Wisconsin, Madison, WI 53706 and Yale Law School, New Haven, CT 06520; Gale: Department of Economics, Georgetown University, Washington, DC 20057. We thank three anonymous referees for numerous valuable comments.

[1] The following quotation makes this point: "Critics maintain that high campaign costs force candidates to devote an inordinate amount of time to raising money. They also hold that special interest groups seeking to exercise influence by satisfying the candidates' need for campaign funds threaten the integrity of the election and governmental processes." (Herbert E. Alexander and Monica Bauer, 1991 pp. 1–2.)

[2] The empirical evidence of a link between campaign contributions and roll-call votes in the House of Representatives and Senate is mixed (Levitt, 1995). There is evidence that lobbyists' influence is felt before legislation reaches the floor, however. Richard L. Hall and Frank W. Wayman (1990) examined committees of the House of Representatives, finding a significant relationship between campaign contributions and members' efforts to shape legislation at the committee stage. Thomas Romer and James M. Snyder, Jr. (1994) found a significant relationship between committee assignments and political action committee (PAC) contributions. In particular, they found that seniority is rewarded, suggesting that contributors target influential members. John R. Wright (1990) studied committee voting and found that campaign contributions facilitate subsequent lobbying.

[3] In one legendary case, five senators met with officials of the Federal Home Loan Bank Board on behalf of a banker who had contributed $1.3 million to the senators and their parties (Alexander, 1991 pp. 116–17).

[4] Recent legislation restricted the types of gifts that members of Congress may accept (*Congressional Quarterly*, 1995a).

[5] Mandatory spending limits were struck down by the Supreme Court in *Buckley v. Valeo*, 424 U.S. 1 (1976). The FECA was then amended to incorporate public funding for presidential candidates who voluntarily accept spending limits. Some recent proposals include voluntary spending limits for congressional campaigns and stricter contribution limits (*Congressional Quarterly*, 1996).

[6] Lobbying organizations provide a large and growing fraction of total campaign contributions. For example,

The next section presents the model and describes the equilibrium when lobbyists are unconstrained. We then solve for the equilibrium when lobbyists face a cap on individual expenditures. When a cap constrains the high-valuation lobbyist, a lobbyist with a lower valuation for the political prize becomes relatively more aggressive. As a consequence, total lobbying expenditures may rise. Since the high-valuation lobbyist's probability of winning the prize drops, the cap reduces total surplus if private and social valuations coincide. Concluding remarks are contained in the final section.

I. The Model

Two risk-neutral lobbyists seek a political prize.[7] The prize could be a government contract, a military base, or a license to produce a good or service. An incumbent politician determines who will receive the prize. Ethics legislation prevents the open sale of political prizes, so the politician will award the prize to the lobbyist who spends more. We do not model the politician's objective function explicitly, but two interpretations of her behavior are possible.[8] First, the politician may be self-interested. A self-interested politician wishes to extract rents from the lobbyists. Although she cannot sell the prize openly, the politician may accept campaign contributions or in-kind contributions.[9] Second, the politician may be benevolent. In this case, the politician wishes to award the prize to the lobbyist who will add more to social welfare. A benevolent politician will award the prize to the lobbyist who spends more, if she does not know the individual valuations, since a lobbyist with a higher valuation will spend more, on average.[10] (Recent legislation requires lobbyists to disclose their lobbying expenditures for each issue on which they lobby members of Congress.)[11]

Gordon Tullock (1980) considered lobbying in a setting where an individual's probability of winning a political prize depended directly on his lobbying expenditures.[12] The case known as the "all-pay auction" has been analyzed by Arye L. Hillman and John G. Riley (1989) and Michael R. Baye et al. (1993, 1996). In an all-pay auction, bidders submit nonnegative bids simultaneously and the prize is awarded to the highest bidder. The novel feature is that all bidders pay their bids, which is appropriate here since a lobbyist's contributions are not typically returned if his efforts are unsuccessful. The all-pay auction is also appropriate for other rent-seeking games such as labor-market tournaments, as well as for research and development contests.

We analyze the all-pay auction when bidders face an exogenous cap on bids. In keeping with the all-pay terminology, we refer to the politician as "the seller" and to the lobbyists as "the bidders." Bidder i's valuation of the prize is v_i, and $v_1 > v_2 > 0$.[13] If bidder i wins with a bid b_i his payoff is $v_i - b_i$, whereas his payoff is $-b_i$ if he loses. (If they tie, the bidders are equally likely to win.) The rules of the game and the payoffs are known by the bidders, who maximize their individual expected payoffs. We look for a Nash equilibrium in

PACs contributed nearly half of all money spent by incumbents in the 1992 elections for the House of Representatives (Levitt, 1995).

[7] Another interpretation of the model is that the lobbyists are candidates for political office, and the candidate who spends more will win.

[8] A complementary view is that lobbying is directly informative (see David Austen-Smith and Wright [1992] or Austen-Smith [1995]). The information might concern the impact on constituents of pending legislation, and it could take the form of technical research or public opinion surveys, for example.

[9] Even if the politician wished to hold a standard auction rather than an all-pay auction, it might be difficult to commit to return contributions from unsuccessful lobbyists, or for lobbyists to commit to make a contribution if awarded the prize.

[10] In Robert W. Helsley and Arthur O'Sullivan (1994), contributions to a referendum campaign signal the private valuations of the competing lobbies, so citizens' votes depend on which lobby contributes more to the referendum campaign.

[11] See *Congressional Quarterly*, 1995b.

[12] Tullock supposed that lobbyist i's probability of winning the political prize is $b_i^R/(\Sigma_{j=1}^n b_j^R)$, where b_j is the amount lobbyist j spends and R is a constant. In the all-pay auction, $R = \infty$.

[13] In the case of a political prize that may not be awarded until after the next election, the valuations could incorporate the probability that the politician will be reelected.

bidding strategies. The bids accrue to the seller.

Absent a cap on bids, bidder 1 wins the prize with probability $1 - (v_2/2v_1) > 1/2$ in equilibrium and the seller's expected revenue is $v_2(v_1 + v_2)/2v_1 < v_2$.[14] Bidder 2 will bid less than v_2, since he forfeits his bid even if he loses. Bidder 1 will take advantage of bidder 2's passivity by bidding less than v_2 himself, resulting in expected revenue strictly below v_2.[15] A cap on bids can attenuate bidder 1's ability to "preempt" bidder 2, which may increase bidding competition and raise the seller's expected revenue.

II. Equilibrium with a Cap on Bids

We now show the impact of an exogenous cap on bids. Let m denote the maximum allowable bid. We consider $m < v_2$ since a larger cap has no effect. There is a unique equilibrium in essentially all cases. That is, there is a unique pair of cumulative distribution functions for equilibrium bids. We focus on these cases first, followed by the nongeneric cases of $m = v_1/2$ and $m = v_2/2$. For small values of m, the equilibrium is in pure strategies. For larger values, it is in mixed strategies, although it still differs qualitatively from the equilibrium in the absence of a cap. (There are mass points at bids other than zero and there are gaps in the support of the equilibrium bids.) Let $F_i(z) \equiv \text{prob}(b_i \le z)$ denote the cumulative distribution function for bidder i's bids in equilibrium.

Three lemmas provide necessary conditions for the distributions of equilibrium bids. The first lemma, which is stated without proof, shows that there cannot be mass points in the interval $(0, m)$. A sketch of the proof follows. If both bidders have mass points at a particular bid between zero and m, then each has an incentive to move the mass higher, since the (conditional) probability of winning would then jump up. (The same argument holds if both have mass points at zero.) If exactly one bidder has a mass point at the bid, then the other bidder will not place density immediately below that bid, since it would be preferable to move such density above the mass point. But it is profitable to move the mass lower if the other bidder has no density just below the mass point. Since there is an incentive to change bids in both cases, there cannot be mass points in $(0, m)$ in equilibrium. Proposition 1 of Hillman and Riley (1989) contains a proof for the case without a cap on bids (i.e., $m = \infty$).

LEMMA 1: *Neither bidder has a mass point at any bid $b \in (0, m)$. At most one bidder has a mass point at zero.*

An implication of Lemma 1 is that bidder i's expected payoff from a bid $b \in (0, m)$ is $v_i F_j(b) - b$, since there is zero probability that bidder j will also bid b. Another implication is that there cannot be a pure-strategy equilibrium here unless both bidders bid m. We now determine the lower limit of bids made in equilibrium. The proofs of the next two lemmas are in the Appendix.

LEMMA 2: *If $m \in (v_2/2, v_2)$, both bidders have an infimum bid of zero. If $m < v_2/2$, both have an infimum of m.*

Lemma 2 implies that the equilibrium bids are $b_1 = b_2 = m$ when $m < v_2/2$. We now show that there is a gap in the set of possible equilibrium bids when $m \in (v_2/2, v_2)$, and that both bidders have mass points at the cap.

LEMMA 3: *Suppose that $m \in (v_2/2, v_2)$. There exists a constant b' such that both bidders place nonzero density on every $b \in (0, b']$ and zero density on every $b \in (b', m)$. Both bidders have mass points at m.*

The lemmas provide necessary conditions for equilibrium distribution functions. We now

[14] An implication of Lemma 1, which follows, is that the equilibrium is in mixed strategies. Thus, bidder 2 has a nonzero probability of winning.

[15] Suppose that bidder 1 wins with (*ex ante*) probability $\pi < 1$. By the standard revealed preference argument, his expected payment, e, satisfies $\pi v_1 - e \ge v_1 - v_2$ since he could win with probability one by bidding v_2. Thus, $e \le \pi v_2 - (1 - \pi)(v_1 - v_2) < \pi v_2$. At the same time, bidder 2 will not pay more than $(1 - \pi)v_2$ for a probability $(1 - \pi)$ of winning, so the seller's expected revenue is strictly below v_2. By contrast, if the seller could hold an oral auction, she could raise revenue of v_2. This last observation coincides with our result that changing the structure of the auction by imposing a cap may increase expected revenue.

find the unique pair that satisfies these conditions. Consider $m \in (v_2/2, v_2)$. We first determine the distribution functions that make the bidders indifferent among all bids in $(0, b'] \cup \{m\}$, as required by Lemma 3. We then find the equilibrium value of b'.

Since bidder 1 must be indifferent among all bids in $(0, b'] \cup \{m\}$, each bid in that set must yield the same expected payoff. That is,

$$(1)\quad v_1F_2(b) - b = v_1[F_2(b') + (1 - F_2(b'))/2] - m,$$

for all $b \in (0, b']$. The left-hand side gives the expected payoff from bidding $b \in (0, b']$, while the right-hand side corresponds to a bid of m. (When bidder 1 bids m, there is probability $1 - F_2(b')$ that bidder 2 also bids m. The tie is broken in bidder 1's favor with probability $1/2$.) Similarly, a bid $b \in (0, b']$ yields bidder 2

$$(2)\quad v_2F_1(b) - b = v_2[F_1(b') + (1 - F_1(b'))/2] - m.$$

We now use (1) and (2) to show that bidder 2 has mass at zero. Straightforward algebra implies

$$(3)\quad v_1[1 - F_2(b)] = v_2[1 - F_1(b)],$$

for all $b \in (0, b']$.[16] Lemma 1 states that the bidders cannot both have mass points at zero, so either $F_1(0) = 0$ or $F_2(0) = 0$. Since $v_1 > v_2$, (3) implies $1 - F_2(0) < 1 - F_1(0)$, so $F_1(0) = 0$ and $F_2(0) = 1 - (v_2/v_1)$.[17]

The distribution functions can now be specified for bids above zero. Bidder 2's equilibrium expected payoff is zero, since a bid of zero yields $v_2F_1(0) = 0$, so (2) implies

$$(4)\quad v_2F_1(b) - b = 0,$$

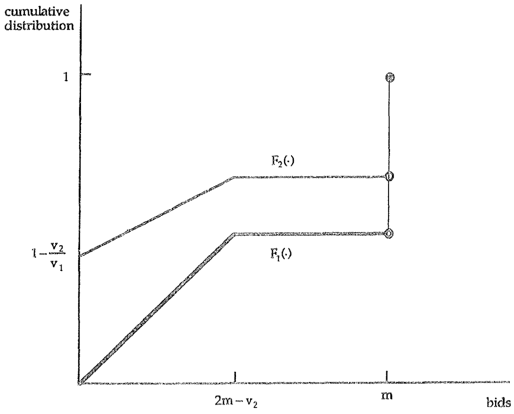

FIGURE 1. EQUILIBRIUM DISTRIBUTION FUNCTIONS

for all $b \in (0, b']$. Thus, bidder 1's distribution function satisfies $F_1(b) = b/v_2$ in that range. Lemma 3 implies $F_1(b) = F_1(b') = b'/v_2$ for $b \in (b', m)$. Finally, $F_1(m) = 1$, by definition.

Bidder 1's equilibrium expected payoff is $v_1F_2(0) = v_1 - v_2 > 0$.[18] A bid $b \in (0, b']$ therefore gives bidder 1 an expected payoff of

$$(5)\quad v_1F_2(b) - b = v_1 - v_2,$$

so $F_2(b) = 1 - (v_2 - b)/v_1$ for $b \in [0, b']$. Finally, $F_2(b) = 1 - (v_2 - b')/v_1$ for $b \in (b', m)$. Equation (2) now implies that $b' = 2m - v_2$. The equilibrium distribution functions are graphed in Figure 1.

All bids made by bidder 1 yield an expected payoff of $v_1 - v_2$, by construction, while all bids made by bidder 2 yield zero. All other feasible bids are inferior to a bid of b' since a bid in (b', m) wins with the same probability as a bid of b', but is more costly. Thus, we have found the equilibrium bidding strategies.

[16] Rearranging (1) yields $m - b = v_1[(1 + F_2(b'))/2] - v_1F_2(b)$, so $m - b' = v_1[(1 - F_2(b'))/2]$. Adding these two equations yields $2m - b - b' = v_1[1 - F_2(b)]$. Repeating the exercise for bidder 2 yields $2m - b - b' = v_2[1 - F_1(b)]$.

[17] Since (3) holds only for $b > 0$, we use the fact that $\lim_{b\downarrow 0}F_i(b) = F_i(0)$.

[18] This holds since an infinitesimal bid gives him an expected payoff of $\lim_{b\downarrow 0}v_1F_2(b) - b = v_1F_2(0) = v_1 - v_2$. Absent a cap, bidder 1 could guarantee a victory by bidding v_2, again yielding $v_1 - v_2$. The cap prevents us from making that direct inference here.

We can now determine the seller's expected revenue. Note first that bidder 1's (*ex ante*) probability of winning the prize is

$$\text{(6)}\quad \int_0^{b'} \frac{1}{v_2} F_2(b)\,db + \left[1 - \frac{b'}{v_2}\right] \times \left[F_2(b') + \frac{1 - F_2(b')}{2}\right] = 1 - (v_2/2v_1).$$

The integral gives the probability that bidder 1 wins, conditional on bidding in $(0, b']$. [The density is $1/v_2$, and a bid b wins with probability $F_2(b)$.] The second term corresponds to the case in which he bids m. Bidder 1's expected payment is the difference between his gross and net expected payoffs:

$$\text{(7)}\quad v_1[1 - (v_2/2v_1)] - [v_1 - v_2] = v_2/2.$$

Using the same approach, bidder 2's expected payment can be expressed as $(v_2/v_1)(v_2/2)$. The seller's expected revenue is the sum:

$$\text{(8)}\quad v_2(v_1 + v_2)/2v_1.$$

Now consider $m < v_2/2$. Lemma 2 implies that both bidders must bid m in any equilibrium. Bidding m is clearly equilibrium behavior. If $b_i = m$, then $b_j = m$ yields an expected payoff of $v_j/2 - m > 0$. A higher bid is not feasible while a lower bid loses with probability one, yielding a nonpositive expected payoff. In equilibrium, each bidder wins with probability $1/2$, and the seller's expected revenue is $2m$. We now summarize the results.

PROPOSITION 1: *If* $m \in (v_2/2, v_2)$, *bidder* 1 *wins with probability* $1 - (v_2/2v_1)$, *and the seller's expected revenue is* $v_2(v_1 + v_2)/2v_1$. *If* $m < v_2/2$, *bidder* 1 *wins with probability* $1/2$, *and the expected revenue is* $2m$.

A cap $m \in (v_2/2, v_2)$ leaves the seller's expected revenue the same as it is without a cap. If $m \in (v_2(v_1 + v_2)/4v_1, v_2/2)$, the expected revenue is $2m > v_2(v_1 + v_2)/2v_1$, so it exceeds the revenue without a cap. The cap is small enough here to remove bidder 1's ability to preempt, but it is large enough that the increase in bidder 2's aggressiveness outweighs the decrease in bidder 1's. For lower m, the expected revenue is strictly lower with a cap than without. Expected revenue is graphed in Figure 2, as m varies.

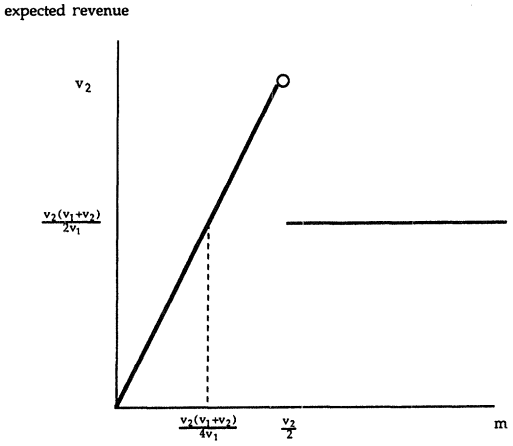

FIGURE 2. EXPECTED REVENUE

Total surplus is (weakly) lower with a cap than without. When $m > v_2/2$, bidder 1's probability of winning is the same as without the cap, so total surplus is unchanged. When $m < v_2/2$, however, bidder 1 wins with probability $1/2 < 1 - (v_2/2v_1)$, making total surplus strictly lower than without a cap.

We now consider the nongeneric cases of $m = v_1/2$ and $m = v_2/2$. If $m = v_2/2$, then there is an equilibrium in which $b_1 = b_2 = m$. There is also a continuum of equilibria in which bidder 2 places mass of v_2/v_1 or more on m and the remainder on zero, while bidder 1 always bids m.[19] If $m = v_1/2$, then there is an equilibrium of the form described in Lemma 3, but there is also an equilibrium in which bidder 1 randomizes between zero and m, while bidder 2 always bids m.[20]

[19] The proof of Lemma 2 showed that an infimum bid in $(0, m)$ was inconsistent with equilibrium, so only zero and m can be infimum bids here. The bidders can have different infimum bids only if one always bids m while the other randomizes between zero and m.

[20] The latter equilibrium requires $m = v_1/2 \leq v_2$, or else bidder 2 is not optimizing. (The cap does not bind if $m =$

The qualitative results from these cases mirror the earlier results. Bidder 1's probability of winning is lower in these equilibria than in the case without a cap, so total surplus is again lower. The equilibria for $m = v_2/2$ again show that expected revenue can be higher with a cap than without.

III. Concluding Remarks

We have shown that an exogenous cap on bids in an all-pay auction (weakly) reduces the probability that the high-valuation bidder wins and increases the seller's expected revenue. When lobbying is seen as an all-pay auction, the results imply that limits on individual expenditures may increase total expenditures and lower total surplus. A cap on campaign contributions may therefore have the perverse effect of increasing aggregate contributions while lowering total surplus.

This paper also contributes to auction theory. We have characterized the equilibrium of the all-pay auction in the presence of an exogenous cap on bids. The results are applicable to a range of contests in which a limit is imposed on effort or expenditure, or in which contestants are constrained because of limited endowments. For instance, caps may increase total expenditures and lower total surplus in a war of attrition that would exhibit preemption in the absence of caps.[21] Limiting individual expenditures on research and development could increase total expenditures, and shorten the expected time to innovation. We conclude with a further discussion of robustness and implications of the results.

A. *Additional Bidders*

Suppose that there are $n > 2$ bidders, with valuations $v_1 > v_2 > \cdots > v_n$. In the absence of a cap, only the two bidders with the highest valuations make nonzero bids, so the expressions for expected revenue and total surplus are unchanged (Hillman and Riley, 1989; Baye et al., 1993, 1996). Now suppose that there is a cap satisfying $v_k/k > m > v_{k+1}/(k+1)$ for some $k < n$. There is an equilibrium with expected revenue of km. Bidding m gives a strictly positive expected payoff to bidders $1, 2, \ldots, k$. Any other feasible bid loses with probability one. At the same time, bidders $k+1, k+2, \ldots, n$ have no incentive to submit a nonzero bid.[22] Thus, expected revenue may again rise relative to the case without a cap. Total surplus is strictly lower than without a cap.

B. *Incomplete Information*

The results do not rely crucially on the assumption of complete information. Rather, the *ex ante* asymmetry in valuations generates the properties discussed above. While explicit handicapping of the high-valuation bidder may increase expected revenue in a first-price auction (Roger B. Myerson, 1981), a *symmetric* cap has that effect here. Thus, symmetric limits on effort or expenditure can substitute for handicapping of favorites in tournaments and contests.

C. *Socially Wasteful Lobbying*

Lobbying expenditures take forms other than campaign contributions. For example, lobbyists spend money on public opinion surveys, and on print, radio, and television advertisements. They spend money encouraging citizens to participate in letter-writing campaigns. They also make in-kind contributions to politicians. Moreover, the value that a politician places on an in-kind contribution may be less than the expenditure involved. These observations suggest that not all lobbying expenditures accrue to politicians.

$v_1/2 > v_2$, so this case does not arise.) This equilibrium is unappealing in the sense that mass is put on a bid that is weakly dominated. Bidder 1 is indifferent between zero and m, but if bidder 2 might tremble and not bid m, then a bid of m would strictly dominate a bid of zero for bidder 1. The same point holds for the equilibria in which bidder 2 randomizes between zero and m.

[21] These preemption results are discussed in Drew Fudenberg and Jean Tirole (1991), for example.

[22] The identity of the active bidders is not uniquely determined if $v_{k+1}/k > m > v_{k+1}/(k+1)$, since there is an equilibrium in which bidder $k+1$ bids m while bidder k bids zero.

Our results do not depend critically on the assumption that bids accrue to the seller. Suppose that a fraction τ of each dollar spent on lobbying is effectively wasted, $0 < \tau < 1$. In the case of a political campaign, contributions enhance the politician's reelection chances, but the value to her of a contribution equal to b is only $(1 - \tau)b$. With two bidders and a cap $m < v_2/2$, expected revenue is $2(1 - \tau)m$, and total surplus is $(v_1 + v_2)/2 - 2\tau m$. There are now two ways that the cap can lower total surplus. In addition to increasing the probability that the prize goes to the low-valuation bidder, the cap may also increase the deadweight loss associated with the wasting of resources. In the region where the cap increases expected revenue, the amount of waste is higher with the cap than without, since waste increases with total expenditures.

D. *Divergence of Private and Social Valuations*

Whether the private and social valuations of the political prize are congruent in any particular context is important. Suppose that one bidder represents a corporation or a closely held business while the other represents a diffuse group, such as consumers. Free-riding among members of the latter group may lead to an understatement of their individual valuations and a consequent lowering of their probability of winning. Imposing a cap and increasing the group's probability of winning may therefore increase total surplus. Similar points hold if the groups differ in both valuations and costs of lobbying.

E. *Empirical Implications*

Aggregate spending on congressional races has doubled since passage of the FECA, which placed controls on campaign contributions. While consistent with the prediction of this paper, such evidence is not definitive since there have been other developments in the intervening years. It is also unclear how tightly the contribution limits bind since there are indirect ways to contribute. (For example, "soft money" can be funneled to political parties.) Thus, a careful empirical analysis is needed.

There are related studies the conclusions of which are consistent with the predictions of this paper. Several researchers have found that close electoral contests induce greater contributions (see James F. Herndon, 1982; Keith T. Poole et al., 1987, for example). To the extent that caps on campaign contributions make elections close, this finding is consistent with the prediction of our paper.

APPENDIX

PROOF OF LEMMA 2:

Let $b^* \equiv \inf\{z \mid F_1(z) > 0\}$ denote the infimum of bidder 1's bids. We first show that only zero or m can be infimum bids in equilibrium. Suppose instead that bidder 1's infimum bid is $b^* \in (0, m)$. If bidder 2 makes a bid in $(0, b^*)$, he loses with probability one. Since a bid of zero is better, bidder 2 must have zero density in $(0, b^*)$.[23] This means that bidder 1 could profitably move density in $(b^*, b^* + \varepsilon^*)$ arbitrarily close to zero. For $b \in (b^*, b^* + \varepsilon^*)$, the payment would drop by b. The probability of winning would drop by only $F_2(b) - F_2(0) = F_2(b) - F_2(b^*) < F_2(b^* + \varepsilon^*) - F_2(b^*)$, however. This last term is of order ε^*, by Lemma 1. It follows that moving the density raises bidder 1's expected payoff, for some $\varepsilon^* > 0$. Since a profitable deviation exists, an infimum bid of $b^* \in (0, m)$ cannot occur in equilibrium. The symmetric argument shows that bidder 2 cannot have an infimum in $(0, m)$ either, so only zero and m are possible infimum bids in equilibrium.

The remainder of the proof comprises two cases. Suppose first that $m \in (v_2/2, v_2)$. We employ a proof by contradiction to show that both bidders have an infimum of zero. Suppose instead that $b^* = m$, which implies that bidder 1 bids $b_1 = m$. Bidder 2 will bid zero or m, or he will randomize between the two, since a bid of zero strictly dominates any $b \in (0, m)$. Bidding zero is inconsistent with

[23] This condition on the density function holds for *almost every* bid in the interval $(0, b^*)$. That is, it can be violated on a set of measure zero without changing the result. In the interest of brevity, we leave this qualification understood throughout the paper.

equilibrium since $b_1 = m$ is not optimal if $b_2 = 0$. Bidding m or randomizing between zero and m can only be optimal for bidder 2 if $v_2/2 - m \geq 0$, since a bid of m results in a tie. This restriction on m contradicts $m \in (v_2/2, v_2)$, so $b^* = m$ cannot occur in equilibrium here. An analogous argument shows that bidder 2 cannot have an infimum bid of m, so the common infimum is zero.

Now suppose that $m < v_2/2$. We employ a proof by contradiction to show that both bidders have an infimum of m. Bidding m guarantees at least a tie, so bidder i must receive an expected payoff of at least $v_i/2 - m > 0$. Suppose that bidder i has an infimum bid of zero. Since his infimum is zero, a bid near zero must be as good as a bid of m for bidder i. But if bidder j does not have mass at zero, then bidder i receives less than $v_i/2 - m$ if he bids near zero. (The probability that bidder i wins would be arbitrarily small for bids that are arbitrarily close to zero.) Bidder j must therefore have mass at zero. Since bidder j's infimum is also zero, the same argument implies that bidder i must have mass at zero. The bidders cannot both have mass at zero, by Lemma 1, so the infimum must equal m for both bidders.

PROOF OF LEMMA 3:

We first show that both bidders have mass points at m. Lemma 2 shows that the common infimum is zero, while Lemma 1 shows that at least one bidder has no mass at zero. Suppose, in particular, that bidder i does not have mass at zero. Then, if bidder j bids arbitrarily close to zero, his expected payoff is strictly below $v_j - m > 0$. Since his infimum is zero, a bid near zero must be as good as a bid of m for bidder j. But if bidder i does not have mass at m, then a bid of m would yield bidder j an expected payoff of $v_j - m$. Bidder i must therefore have mass at m. We conclude that at least one bidder has mass at m, since at least one bidder has no mass at zero.

Suppose, in particular, that bidder i has mass $\alpha > 0$ at m. If bidder j has nonzero density in $(m - \varepsilon', m)$, he could profitably move it to m, for some $\varepsilon' > 0$. For $b \in (m - \varepsilon', m)$, the payment would rise by only $m - b < \varepsilon'$, but the probability of winning would rise by at least $\alpha/2$, since bidder j would now tie if bidder i bids m. Since moving the density up raises bidder j's expected payoff, bidder j must have zero density in $(m - \varepsilon', m)$. If bidder j has no mass at m, then bidder i could profitably take mass from m and move it lower. We conclude that both bidders have mass points at m.

The presence of mass points at m for both bidders implies that both bidders have zero density in $(m - \varepsilon'', m)$, for some $\varepsilon'' > 0$. This demonstrates the existence of $b^* \in [0, m]$ such that both bidders have zero density in (b^*, m). Let b' denote the smallest $b^* \in [0, m]$ such that both bidders place zero density on every bid in (b^*, m).[24] We now show that both bidders place nonzero density on every $b \in (0, b']$. By the argument used in the proof of Lemma 2, if bidder i has zero density in an interval $(s, t) \subset (0, b']$, then so must bidder j. But if both bidders have zero density in (s, t), then either bidder could profitably move density from $(t, t + \varepsilon^*)$ down to s, for some $\varepsilon^* > 0$. Thus, both bidders must have nonzero density on every $b \in (0, b']$.

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[24] In fact, $b' \in (0, m)$. The preceding paragraph shows that $b' \neq m$. If $b' = 0$, then both bidders must have mass points at zero, by Lemma 2. This contradicts Lemma 1, so $b' \neq 0$.

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